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VIBRATIONS SYMPTOMS IN STATE DESTRUCTION OF BUILDING CONSTRUCTION

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ABSTRACT

Building objects are subject to large dynamic loads clearly reflected by generated vibration processes. The vibrations may affect structures serviceability state by lowering comfort of persons working there as well as possibility of reaching the hazardous level to structures safety. Judging the necessity of improving the quality assessment methods of building structures for purposes of state estimation as well as safety factors for brick structures, the authors of this work undertook an attempt to investigate destruction process of selected brick object by using the method of experimental modal analysis.

Key words: modal analysis, natural vibration frequency, stabilization diagram, structural vibrations

INTRODUCTION

Modern building structures, production of silent-running machines and devices are associated with a high precision level of their manufacturing and appropriate selection of materials that greatly influence their quality, reliability and durability (Ibrahim & Mikulcik, 1977; M. Żółtowski, 2007, 2014a).

In investigating real systems (structures, buildings, machines, devices) the main problem is to determine quantity of energy stored, dissipated and transmitted by particular elements of the systems. Knowledge of the quantities serves to assessing material effort, fatigue, diagnostic investigations as well as predicting noise levels, and also to facilitate designing system's elements, e.g. vibration isolation (Shih, Tsuei, Allemang & Brown, 1988; Brunarski, 1996; Batel, 2002; Żółtowski, Łukasiewicz & Kałaczyński, 2012; Żółtowski, 2014b).

Development of measurement methods, especially those for measuring energy quantities, has substantially extended possibility of research on sound radiation by structures as well as made it possible to calculate sound power radiated to a remote field on the basis of close-field measurements (Bishop & Johnson, 1980; Natke & Cempel, 1997; Pickrel, 2002; Vold, Kundrat, Rocklin & Russell, 2010; Żółtowski, 2014b).

Contemporary structural dynamics in building engineering makes use of various research tools from the state identification area such as: boundary element method, finite element method and modal analysis methods, which enables – by modelling and investigating state changes – better behavior understanding of complex structures, perform their optimization during design process and assess their current, often hazardous, states (Wiliams, Crowley & Vold, 1985; Peeters & Ventura, 2001; Formenti & Richardson, 2002; Żółtowski, Żółtowski & Castaneda, 2013).

It is necessary to improve methods for dynamic characteristics of structures research especially those exposed on large dynamic loads. New materials and technology methods have been introduced to building engineering as well as novel structural solutions make

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it possible to increase productivity and quality of products, however they are accompanied with large, often dangerous dynamic loads ((Uhl, 1997; Żółtowski, 2005, 2007; Żółtowski et al., 2012; Żółtowski et al., 2013).

In building engineering, vibrations – a process which accompanies any motion – may be considered in the categories of noxiuos, favor able or information containing vibrations. Vibrations are primary process and their (secondary) effect is acoustic signal in the form of longitudinal sound wave. Vibration and noise processes form the basis for a scientific research area – vibroacoustic. Modern building structures are accompanied by vibroacoustic phenomena which endanger people, environment, and their products (Ibrahim & Mikulcik, 1977; Richardson, 1997; Uhl, 1997; Bischoff, Wall, Bletzinger & Ramm, 2004).

In most cases met in practice, analyses of dynamic properties are performed on the basis of analysis of structural model behavior. Quality of the analysis depends on credibility of the model, which is measured by means of conformation of the object's behavior and the model, both subject to disturbances of the same kind. Structural model may be built in the process of analytical transformations used for description of system's dynamics or on the basis of results of experiments performed on a real object (Brandt, 1999; Pickrel, 2002; Peeters & Ventura, 2003; M. Żółtowski, 2011a).

Analysis of structure dynamic properties are carried out mainly by examining the behavior of dynamic model of a given structure, which is realized by using analytical description of quantities, which characterize system's dynamics, or experimental methods directly applied to real objects (Uhl, 1997; M. Żółtowski, 2011b).

Modal analysis is widely used for investigating degradation state and fault location, modification of dynamics of tested structures, description and updating analytical model, as well as monitoring structural vibrations in aircraft and civil engineering. In the subject-matter literature the following notions can be found: modal analysis, experimental modal analysis and operational modal analysis (Natke & Cempel, 1997; Uhl, 1997; Pickrel, 2002; Peeters & Ventura, 2003; M. Żółtowski, 2011a). In the majority of practical applications of modal analysis, a multi-channel experiment and complex calculations connected with the processing of measured signals and estimation of model's parameters, are required.

VIBRATIONS IN DESCRIPTION OF STRUCTURES

Vibroacoustic is a domain of science which deals with any vibration, acoustic and pulsation processes occurring in nature, building engineering, technology, machines, devices, communication and transport means, i.e. in the environment. Among the tasks of vibroacoustic the following may be rated (M. Żółtowski, 2005; Vold et al., 2010):

- the identification of vibroacoustic energy sources which consists in location particular sources within structure of object, machine or environment, determination of their characteristics and mutual relationship, determination of vibroacoustic power as well as character of vibration and sound generation;
- the elaboration of vibroacoustic energy propagation paths in real structures and environment (buildings, machines, objects etc.), theory of energy transmission and transformation, passive and active control means for phenomena, methods for analyzing and testing phenomena at the border area between wave and discrete approach;
- the elaboration of control methods for vibroacoustic energy (emission, propagation) in building structures, machines and environment, and also elaboration of methods for steering the phenomena, that is associated with active methods which are presently under development worldwide;
- the use of vibroacoustic signals for purposes of technical state diagnostics as they constitute a good carrier of information on state of object's destruction as well as technological process under way (vibroacoustic diagnostics);
- the vibroacoustic synthesis of objects, performed to obtain optimum vibroacoustic activity (structural, kinematic, dynamic), which covers synthesis of parameters used in active methods for vibration and noise mitigation, and structural, kinematic and dynamic synthesis of objects and machines;
- the active applications of vibroacoustic energy to realizing various technological processes, beginning from ultrasonic welding and cleaning, transport of materials and machine elements along tech-

nological lines, consolidation of molding sands, shaking out and cleaning castings, ending at consolidation of soils and concretes.

Vibroacoustic process may be presented as:

- generation of time-varying forces acting onto a structure and its environment;
- propagation and transformation of energy in different environment structures;
- sound radiation through material elements of environment.

In analysis of vibroacoustic processes the following is taken into account:

- time and space distribution of run of energy coming from a (primary), source,
- response of a system (structure, liquid) as well as energy transmission through propagating media,
- mutual relations between sources.

The notion of measurement means a process of acquisition and transformation of information about a measured quantity to get – by comparing it with measurement unit – a quantitative result in a form most comfortable to be acquired by human sense organs, its transmission in space or time (recording), mathematical processing or application to steering. To carry out such measurements is necessary for (Bishop & Johnson, 1980; M. Żółtowski, 2011a):

- the determining of time runs of vibrations and their parameters to determine kinds of the vibrations, their characteristic quantities and to perform detail analysis;
- the finding of vibration sources and places of their occurrence;
- the determining of characteristic features of systems (e.g. determining loads during vibrations and their dependence on object's parameters, its shape, dimensions, material properties etc.);
- the minimizing of vibrations harmful for reliable operation of devices and their human operators;
- the determining of harmfulness level of occurring vibrations and the implementing of preventive measures.

In practice, vibration signal is more often used than noise one, due to its easiness of transferring and exactness of measuring (Brunarski, 1996; Vold et al., 2010).

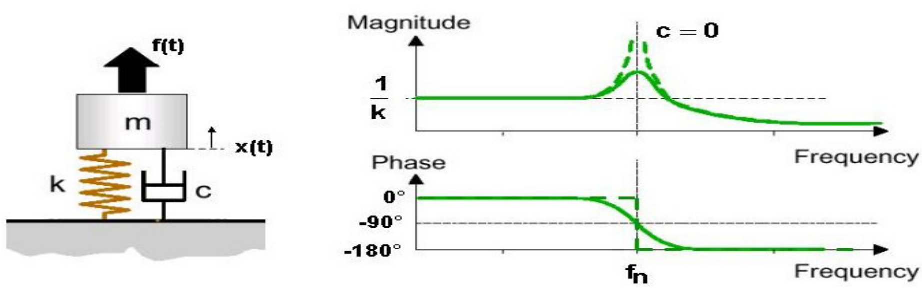
System's vibrations resulting from upsetting state of equilibrium of an object which then moves under action of elastic, gravity or friction forces, are called free vibrations. In one degree-of-freedom (d.o.f.) systems the upsetting of state of equilibrium is characterized by the initial conditions: the initial position (x_0) and initial velocity (v_0). If the system is of one-degree-of-freedom (single mass – m) and linear characteristics of elasticity (k) and damping (c) – Figure 1, and the harmonic excitation force $F(t)$ acts onto it, then its motion equation is expressed by the following formula:

$$m\ddot{x}+c\dot{x}+kx=F(t) \qquad (1)$$

which represents the equation of harmonic vibrations or harmonic oscillator vibrations.

As results from it, natural vibration of one-degree-of-freedom system is entirely determined by natural frequency of vibration. Amplitude of the vibration depends on initial conditions, but natural frequencies and vibration period do not depend on them.

The parameters: a, v, x – are those of vibration process, which convinces that the vibrations properly describe state of structure (Natke & Cempel, 1997).



$$H(s)=\frac{1}{ms^2+cs+k}$$

Fig. 1. One-degree-of-freedom system to perform translation motion

In the low frequency range, building structures can be modelled by means of discrete systems of a few one-degree-of-freedom – and rather often – single one. The discrete system – in contrast to continuous one – is characterized by point distribution of mass, stiffness and damping and dimensions of the elements do not play any role. Number of one-degree-of-freedom determines number of independent coordinates which should be introduced to get unambiguous description of system's motion (number of one-degree-of-freedom is equal to number of mass elements in the system in question). In practice, the system presented in Figure 1 can model:

- the building machine of mass (*m*), seated on shock absorbers (*k*, *c*) and fastened to a big mass foundation;
- the work machine of mass (*m*), seated on shock absorbers (*k*, *c*) and moving along an even road;
- the high building structure (high chimney, masts) under wind action.

Many systems can be preliminarily modelled by using one-degree-of-freedom system, to search for its properties by means of mathematical description and analysis of solutions of equations which describe it. It is possible to investigate system's properties by using the vibration parameters (*a*, *v*, *x*) which – being results of solutions of mathematical description of the model – interchangeably describe the same properties but from the viewpoint of the system's vibration measuring process. In industrial practice it is common to measure vibrations instead to perform complex theoretical considerations.

The use of vibrations for testing quality of building structures results from the following reasons:

- vibration processes reflect physical phenomena occurring in structures (displacements, stresses, fractures), on which degree of their destruction (serviceability) and correct operation depends, that results from character of spreading the vibration process;
- easiness of performing measurements of vibration processes in normal operational conditions of an object without necessity of exclusion it from service and performing special preparation, hence it makes it possible to assess its state of destruction without disassembling the structure;
- vibration processes are characterized by a high speed of information transmission per time unit, defined by Shannon formula:

$$C = F \lg_2 \left(1 + \frac{N_S}{N_Z}\right) \tag{2}$$

  depending on spectrum band of the process (F) and the rate of the useful signal power (N_S) and the disturbing noise power (N_Z);
- vibration processes are characterized by a complex structure of time, amplitude and frequency, which, if only correctly processed, makes it possible to assess state of entire structure as well as its particular elements.

During service of structure, due to occurrence of many external factors (excitations from the side of environment and other structures) and internal factors (ageing, wear, interaction of elements) in the structure take place disturbances of its equilibrium state, which propagate within elastic body, material of which the structure is made. The disturbances are of dynamic character and maintain equilibrium conditions between inertia, elasticity, damping and excitation state. Consequently, it results in energy dissipation of waves, their deflection, reflection and mutual superimposing. Existence of sources and propagation of disturbances cause vibration of structural elements and surrounding environment to occur.

Internal input taken as a set of excitation quantities which determine object's structure (shape, manufacturing quality, clearances etc.) and a way of interaction of its elements is formed in random conditions during manufacturing, that reveals object's random properties in service. External input which determine conditions of interaction between structure elements and other elements of a system (changes of loads, speed, environment impact) is also practically of a random character.

Many possible occurrences of randomness and disturbances result in additional assumptions dealing with inputs and occurring transformations of states of destruction of structures. They concern with assumptions on linearity, stationarity i ergodity of models of objects and processes (Formenti & Richardson, 1982; Shih et al., 1988; M. Żółtowski, 2007, 2011a). As a result of existence of the input and realization of transformation of states, which represent processes occurring in structure, many measurable characteristic symptoms contained in output processes emitted from structure, are obtained. The processes form the basis for elaboration of a signal generation model which determines a way of forming, functioning and changing states of object's destruction (M. Żółtowski, 2011b, 2014b).

The described set of assumptions which leads to a model of generation of signals can be presented in the form of the schematic diagram shown in Figure 2.

The output vibration signal at any reception point can be approximately expressed by the following formula (Natke & Cempel, 1997; Vold et al., 2010):

$$y_k(\theta, r) = \sum_{i=1}^{k} a(k) h_i (t, \theta, r) \cdot [u_i (t, \theta, r) + n_i (t, \theta, r)] \quad (3)$$

where:

$h(*)$ – the impulse transition function, which covers material destruction properties;

$a(k)$ – term which provides different summation weights connected with the reception place (r).

The presented way of interpretation of the output signal (θ, r) is – in the general case of excitations of periodical service objects – correct, but not always so simple as that shown in Figure 3 where is illustrated occurrence of excitations due to random actions of wind onto high buildings, chimneys, towers, and record of relevant response in the form of complex vibration signal.

The output signal received in an arbitrary point of structure is the weighted sum of responses to all elementary events (t, θ, r) which occur always in the same sequence in particular points of the dynamic system of the pulse transition function $h(t, \theta, r)$. The influences sum up together and subject to additional transformation along different reference axes, and a change of signal reception point r is associated also with change of transmittance.

Model of vibration signal transmitting through tested structures or brick wall elements is described practically by FRF function which is determined by means of experimental modal analysis in the form of ratio of vibration excitation force and vibration acceleration amplitude at output. The transmittance $H(f)$ defined as the response-to-excitation ratio is inversion of the FRF function.

The indicated properties of the elaborated model of signal transition through tested materials were further used for assessing changes of degree of degradation of structures or brick wall elements during testing transition of vibration signals through various structures of brick wall elements and segments.

Modal analysis is widely applied to removing damages resulting from vibrations, modifying structure dynamics, updating analytical model or state control, and also used for monitoring vibrations in aircraft

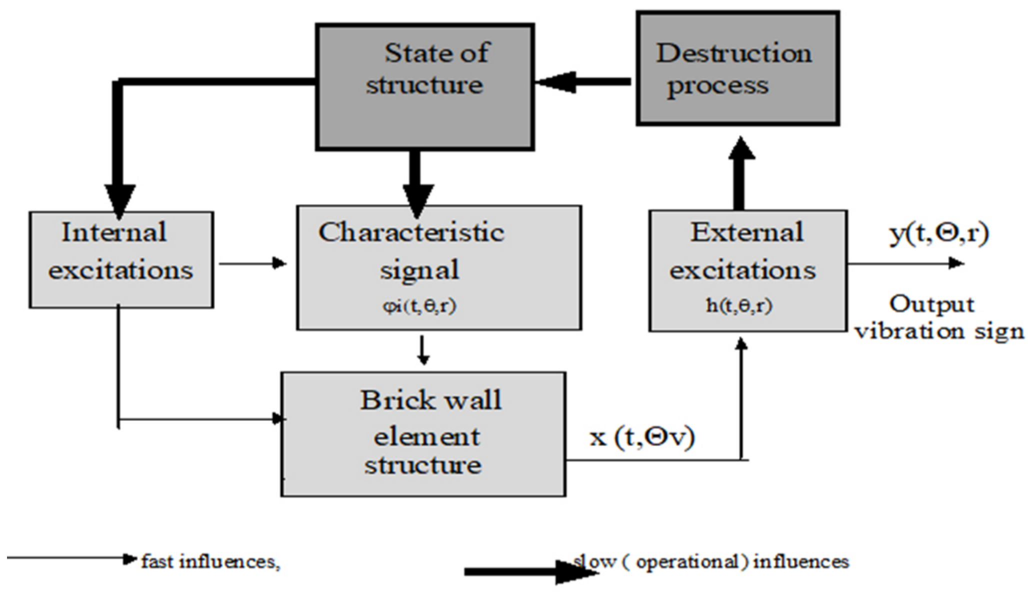


Fig. 2. Model of signal transmission through tested brick wall element (own elaboration)

industry and civil engineering (Ibrahim & Mikulcik, 1977; Uhl, 1997; Pickrel, 2002; M. Żółtowski, 2007).

Theoretical modal analysis is defined as a matrix eigenvalue problem dependent on matrices of mass, stiffness and damping. It requires the eigenvalue problem for an assumed structural model of investigated structure to be solved (Uhl, 1997; M. Żółtowski, 2014a, b). The determined sets of natural frequencies, damping coefficients for the natural frequencies and forms of natural vibrations make it possible to simulate behavior of structure under arbitrary excitations, choice of steering means, structural modifications and other issues.

Analysis of natural frequencies and vectors is obtained on the basis of motion equations (after neglecting terms which contain damping matrix and external load vector). Then the motion equation of natural vibrations obtains the following form:

$$B\ddot{q} + Kq = 0 \tag{4}$$

For one-degree-of-freedom system its solution is as follows:

$$q(t) = \vec{q}\sin(\omega t + \phi) \tag{5}$$

where:

$\vec{q}$ – vector of amplitudes of natural vibrations.

On substitution of equation (5) second derivative to the motion equation the following is obtained:

$$\left(-\overline{\omega}^2 B + K\right)\vec{q}\sin\left(\overline{\omega}t + \phi\right) = 0 \tag{6}$$

The equation is to be satisfied for arbitrary instant (t), then the set of algebraic equations is yielded as follows:

$$\left(K - \overline{\omega}^2 B\right)\vec{q} = 0 \tag{7}$$

$$\left(k_{11} - \omega^2 m_{11}\right)q_1 + \left(k_{12} - \omega^2 m_{12}\right)q_2 + \ldots + \left(k_{1n} - \omega^2 m_{1n}\right)q_n = 0$$

$$\left(k_{21} - \omega^2 m_{21}\right)q_1 + \left(k_{22} - \omega^2 m_{22}\right)q_2 + \ldots + \left(k_{2n} - \omega^2 m_{2n}\right)q_n = 0$$

$$\left(k_{41} - \omega^2 m_{41}\right)q_1 + \left(k_{42} - \omega^2 m_{42}\right)q_2 + \ldots + \left(k_{nn} - \omega^2 m_{nn}\right)q_n = 0$$

This way was produced the set of linear homogeneous algebraic equations, which has non-zero solution only when the condition $det(K - \omega^2 B) = 0$ is fulfilled.

On transformations the n-order polynomial is obtained. Among its roots multifold ones may be present, and the vector built from the set ω^2 of frequencies ordered according to increasing value sequence is called the frequency vector, and the first frequency is called the fundamental one (M. Żółtowski et al., 2013).

$$\overline{\omega} = \left[\overline{\omega}_1, \overline{\omega}_2, \ldots, \overline{\omega}_n\right] \tag{8}$$

The theoretical modal analysis is mainly used in design process, i.e. when it is not possible to perform tests on objects. The traditional experimental modal analysis (EAM) makes use of input (excitation) to output (response) relation and it is measured in order to assess modal parameters consisted of modal frequencies and damping. However, the traditional EAM has some limitations such as:

- in the traditional EAM, artificial excitation is used to measure vibration frequencies;
- the traditional EAM is usually performed in laboratory conditions.

In many cases a real state of degradation may greatly differ from those observed in laboratory environment. In experimental modal analysis the identification

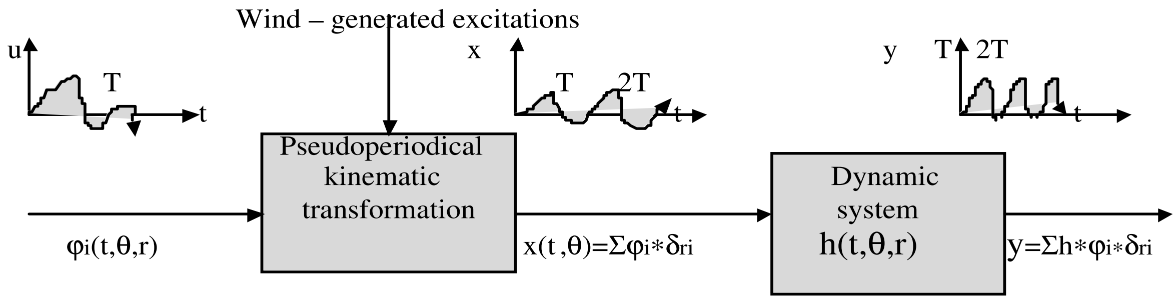


Fig. 3. Transformation of the characteristic signal – $\phi_i(*)$ into the output signal – $y(*)$, considered to be a model of signal generation in objects under environmental excitation (Natke & Cempel, 1997)

experiment consists in exciting object's vibrations at simultaneous measuring excitation force and system's response usually in the form of vibration acceleration amplitude.

RESULTS

For the waveform's extortion measurement, and determination of the most used functions FRF measurement LMS TEST.XPRESS software was used. It's enables to easily perform a modal analysis of brick elements, as well as any other building structures.

During the tests it was able to generate a transfer function of vibration signal by the structure (FRF function). The results are presented in real time in the center of a screen. It's allows visualizing the temporary courses of extortions and the answer (Fig. 4).

From a large group of building materials, a fit and a damaged brick were measured to compare their fitness. Figure 5 shows the results obtained after performing measurements in axis Y, because in brick walls compressive strength can be most destructive. For a better visualization of the investigation results they are shown below separately – 30 times the FRF

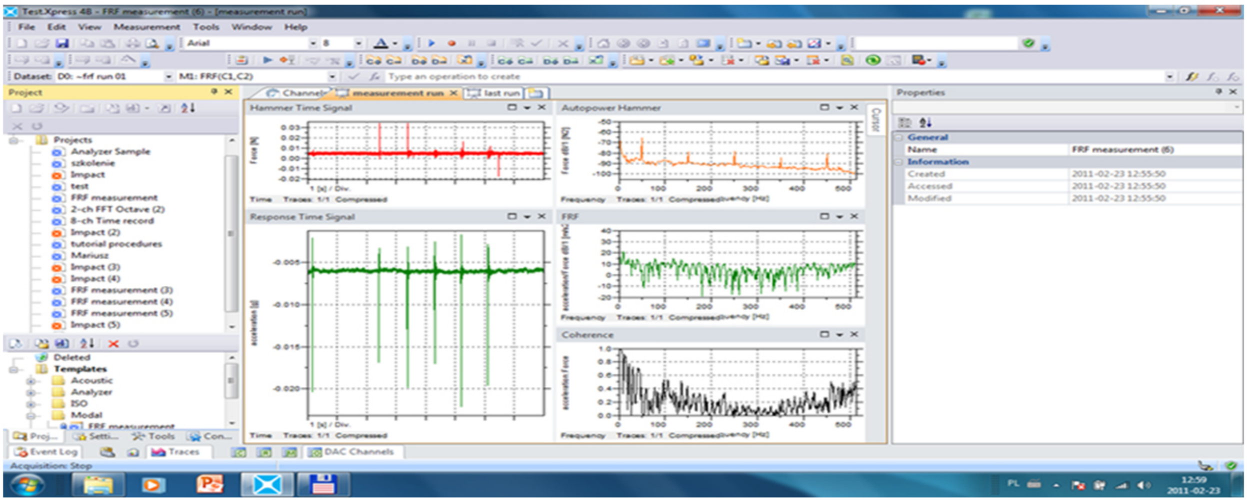


Fig. 4. Example exposition of results of measurement (own study)

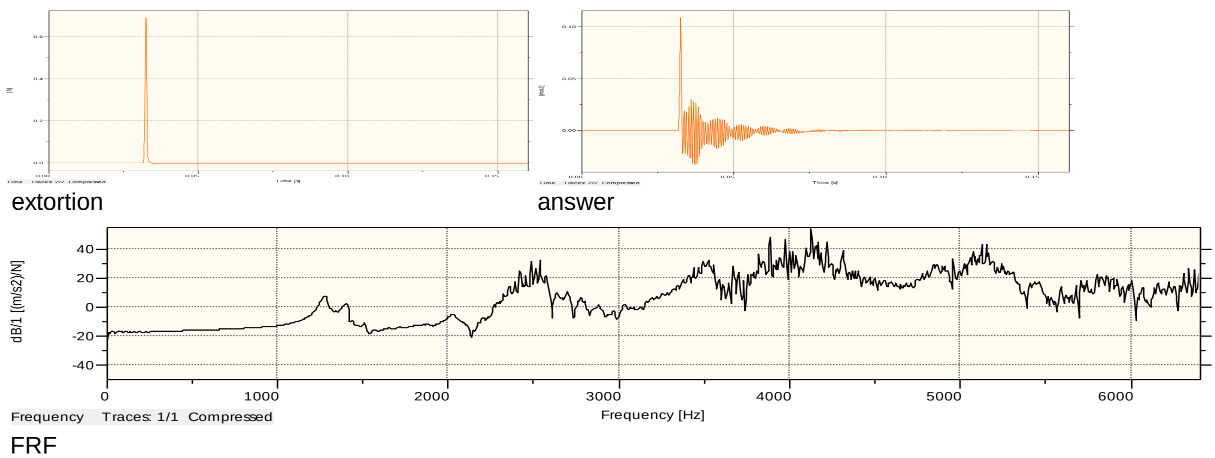


Fig. 5. Composition of measurements results (the temporary course of extortion, temporary course of answer, function FRF) of full bricks in axis Y

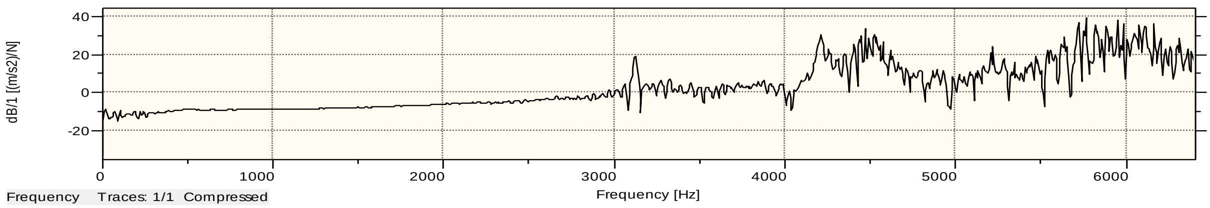


Fig. 6. Composition of FRF functions of 30 full bricks in axis Y

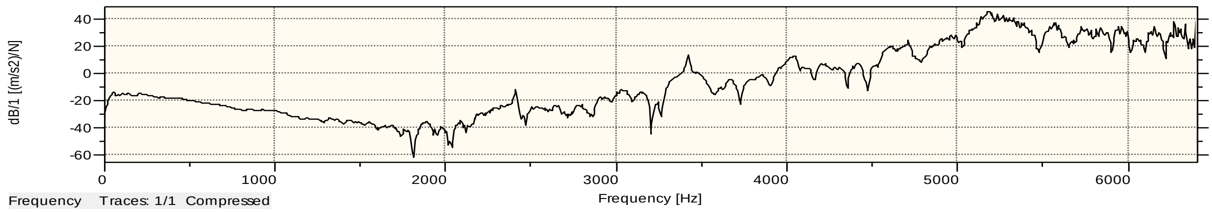


Fig. 7. Composition of FRF functions of 30 destroyed full bricks in axis Y

function for good, and 30 times for destroyed brick element. Figure 5 shows once the extortion, and the answer of signal in time domain, which allows to gain FRF function. Graphic results which shows FRF functions of good, and destroyed bricks measured in axis Y are show in Figures 6 and 7.

CONCLUSIONS

The results point to the fact that it is possible to distinguish between material properties, which has an impact on the ability to distinguish between their mechanical properties. The study also confirmed the usefulness of the LMS test apparatus using operational modal analysis performed on the actual building construction.

By obtaining graphical charts of FRF function, and a later their comparison it is possible to observe their diversity. These charts are different for materials that are in good condition, and damaged, which demonstrates the ability to assessment of the destruction of a brick element.

It practically verified the sensitivity of assessment of modal analysis to degree of brick structure degradation. It becomes possible to determine hazards to a building structure on the basis of examining values of frequencies.

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DRGANIOWE SYMPTOMY W BADANIU STANU DESTRUKCJI

STRESZCZENIE

Obiekty budowlane podlegają dużym obciążeniom dynamicznym, wyraźnie odzwierciedlonym w generowanych procesach drgań. Drgania mogą wpływać na stan użytkowalności konstrukcji, obniżając komfort osób tam pracujących, a także mogą spowodować osiągnięcie poziomu niebezpiecznego dla stabilności konstrukcji. Oceniając konieczność poprawy metod ocen destrukcji konstrukcji budowlanych dla celów oceny stanu, a także współczynników bezpieczeństwa dla budowli, autorzy niniejszej pracy podjęli próbę zbadania procesu niszczenia wybranego obiektu z cegły, stosując metodę eksperymentalnej analizy modalnej.

Słowa kluczowe: analiza modalna, częstotliwość drgań naturalnych, wykres stabilizacji, drgania strukturalne