

TRANSPORT OF MUTUALLY COOPERATING TRIPLETS OF THERMAL FOURIER FLUCTUATIONS THROUGH TWO-PHASE PERIODICALLY LAYERED PARTITION WALL*

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ABSTRACT

The paper analyses boundary effect behaviour in a building partition made of a two-component layered composite. A two-dimensional model of such barrier has been adopted in which the boundary effect behaviour is described by a system of ODE's. It has been proved that, for selected types of fluctuations, a hypothesis formulated in the end of the paper is true, i.e. a partition reacts for the presence of boundary package of fluctuations consisting of one even and two odd fluctuations merely with a typical exponentially damped transport of these boundary fluctuations. Their sinusoidal pulsations in the direction transversal to the periodicity directions are not possible. The exponential damping is maximal for components with very different material properties (values of parameters k_{11}/k_1 and η_1 close to zero). Such situations correspond to a characteristic peak of the graphs included in the paper.

Key words: heat conduction, effective heat conduction, boundary effect behaviour, building partition

INTRODUCTION

Composite materials are often subjected to loads that cause boundary fluctuations in a temperature field and a displacement field. The case of boundary fluctuations in a displacement field concerns, for example, a situation when it is necessary to protect precise electronic devices against harmful ultrasounds. This protection may consist in placing such devices in rooms with composite walls. The effectiveness of this protection depends on the material properties of the composite. If it is a periodic composite, the best damping of displacement field disturbances is observed in the direction perpendicular to the composite periodicity. But a necessity of transmission of fluctuations does not always evoke a reaction of the composite in a form of boundary damping of these fluctuations. It may occur that the reaction is pulsatile. The same behaviour is observed in the problems of heat conduction. In this case, boundary fluctuations in the temperature field are most often caused by a spontaneous formation of a boundary layer in the area of the partition from the outside and inside of the room.

* The article is formatted in one-column layout due to complexity of the text.

It can be assumed that the ambient temperature automatically adjusts to the heterogeneous structure of the composite, creating boundary fluctuations. The boundary fluctuations that arise in this way are usually extinguished inside the wall. In this case, a boundary effect occurs. However, the boundary fluctuations often pulsate. It is the aim of this work to investigate when they pulsate. We observe similar phenomena in the area covered by the earth's tectonic plates (Chelidze et al., 2010a, 2010b; Ponomariw, Lockner, Stroganova, Stanchits & Smirnov, 2010).

Thermal boundary impulses of a temperature field will be treated as impulses superimposed on an average temperature in a region of a repeatable cell of a periodic composite – such attitude toward the pulses is an application of the physical micro-macro hypothesis. This is the basic hypothesis of the so-called tolerance modelling (Woźniak & Wierzbicki, 2000) or rather its extension (Kula, Mazewska & Wierzbicki, 2012; Kula & Wierzbicki, 2015) leading to an accurate description of the phenomena of heat conduction in composites.

The work is an attempt to use a detailed description of the thermal boundary effect behaviour described by Kula (2016). Considerations in this work, as already underlined, concern only the issues of heat conductivity.

The study is limited to the simplest behaviour of thermal conductivity using the Fourier's law of thermal conductivity. The result of this choice is the use of the parabolic equation of thermal conductivity as a physical starting point for the work; precisely, the equivalent reformulation of the thermal conductivity equation for periodic composites, called the surface localization of this equation, will be used. Basing on such way of description of the phenomena of thermal conductivity, it has been stated in the paper that, during transport through the composite area, the periodic boundary fluctuations experience not only an intense exponential damping but also rotational damping – sinusoidal pulsations. The work is devoted to the analysis of these two types of damping. For the first time, the analysis of the boundary effect behaviour with use of the so-called Surface Localized Heat Transfer Equations was undertaken by Kula (2016). A similar issue, but concerning only a pair of Fourier fluctuations (Wodzyński, Kula & Wierzbicki, 2018).

The aim of this work is a determination of the maximum damping coefficient of the mutually cooperating triplets of thermal Fourier fluctuations during their transport through a two-phase periodically layered partition.

The scope of the work is limited to the analysis of the role of a composite building partition as a protection of precise electrotechnical devices against harmful disturbances caused by external physical fields. Such partition is designed to act as a filter for such disturbances.

The subject of the study is the analysis of the type of a reaction of a conductor to the transport of a package (set) of such fluctuations through the area occupied by this conductor (partition). The conductor reacts to the transport of such a package either by an extinguishing (damping) it or by making the transmitted fluctuations oscillate across the conducting partition. These oscillations can be very damaging both to the partition and for devices located in a room surrounded by such partition.

The aforementioned analysis of the conductor's reaction type is formalized in this paper to an analysis of the appropriate Cauchy problem for a system of ordinary differential equations of the second order, which is a mathematical description of the boundary effect behaviour.

SURFACE LOCALIZATION OF HEAT CONDUCTION IN COMPOSITE MEDIA

Heat conduction in periodic composites

The starting point of considerations in this work is the parabolic equation of heat conduction in a form

$$\nabla^T (K \nabla \theta) - c \dot{\theta} = b \quad (1)$$

which describes the heat conduction in a composite partition occupying an area $\Omega \subset R^D$, $2 \leq D \leq 3$, specified as a Cartesian product

$$\Omega = \Omega_d \cdot \Omega_{D-d} \quad (2)$$

where $\Omega_d \subset R^d$ and $\Omega_{D-d} \subset R^{D-d}$, moreover:

$$1^\circ \Omega_d = (0, L), \Omega_{D-d} = (0, \delta_1) \times (0, \delta_2) \text{ if } (d, D) = (1, 3),$$

$$2^\circ \Omega_d = (0, L_1) \times (0, L_2), \Omega_{D-d} = (0, \delta) \text{ if } (d, D) = (2, 3),$$

$$3^\circ \Omega_d = (0, L), \Omega_{D-d} = (0, \delta) \text{ if } (d, D) = (1, 2),$$

for $L_1, L_2, L, \delta_1, \delta_2, \delta > 0$. In Eq. (1):

- $\theta = \theta(y, z, t), y \in \Omega_d \subset R^d, z \in \Omega_{D-d} \subset R^{D-d}, t \geq 0$, denotes a temperature field,
- c denotes a specific heat field,
- k denotes a conductive matrix field.

Moreover, $\nabla \equiv \nabla_d + \nabla_{D-d}$ for $\nabla_d \equiv [\partial / \partial y^1, \dots, \partial / \partial y^d, 0, \dots, 0]^T$ with the last $D - d$ terms equal to zero, as well as $\nabla_{D-d} \equiv [0, \dots, 0, \partial / \partial z^1, \dots, \partial / \partial z^{D-d}]^T$ with the first d terms equal to zero. Both these fields $c = c(\cdot)$ and $k = k(\cdot)$ are functions independent on the temperature field and determined everywhere in Ω_d apart of the places (lines or points, depending on the value of $d = 1$ or 2) separating components of the composite and assuming S values equal to c^1, \dots, c^S and k^1, \dots, k^S , respectively, in the areas occupied by these components. It has been also assumed that the aforementioned functions were obtained in a way that certain periodic scalar fields determined almost everywhere in the whole space R^d were restricted to the area Ω_d . If the space R^d in Eq. (2) is interpreted as a space of directions of periodicity and R^{D-d} as a space of directions perpendicular to the directions of periodicity, then one can acknowledge that Eq. (1) can be interpreted as an equation describing the heat conduction behaviour in the area occupied by a periodic composite whose periodicity is determined by a repeatable cell Δ – thus one deals with a Δ -periodic composite. A diameter $diam(\Delta)$ of this repeatable cell must not be small with relation to a characteristic linear size L of the area Ω occupied by the composite. It can be assumed that the equations used in this work will be controlled by a dimensionless scale parameter $\lambda = diam(\Delta)/L$, because these equations will depend on this scale parameter. Δ -periodicity of the composite means that it exists a σ -tuple $(\mathbf{v}^1, \dots, \mathbf{v}^d)$ (a series of lengths σ) of linearly independent vectors $\mathbf{v}^1, \dots, \mathbf{v}^d \in R^d$ determining d directions of periodicity and having the properties: (i) points $x + k_1 \mathbf{v}_1 + \dots + k_d \mathbf{v}_d, -0.5 < k_1, k_d < 0.5$, cover the whole inside of the cell $\Delta(x) = x + \Delta, x \in R^D$ (ii) $\Delta = \Delta(x_0)$ for a fixed $x_0 \in R^3$, (iii) $c(x + \mathbf{v}) = c(x), K(x + \mathbf{v}) = K(x)$ for every $\mathbf{v} \in \{\mathbf{v}_1, \dots, \mathbf{v}_d\}, x \in R^D$. In this way, a Δ -periodic structure has been introduced into the whole space R^D . Hence, a Δ -periodic net Γ of surfaces separating the composite components exists in the whole space R^D . An averaging $\langle f \rangle(x), x \equiv (y, z)$, of an integrable function f is understood in the whole work as an integration averaging:

$$\langle f \rangle(x) = \frac{1}{|\Delta|} \int_{\Delta} f(\xi) d\xi \quad (3)$$

and it is independent on x if f is a Δ -periodic function.

Decomposition of the temperature field into long-wave and short-wave part

According to the methodology of surface localization, introduce an ε -neighbourhood (an ε -band)

$$o_\varepsilon(\Gamma) = \{x \in R^D : \text{dist}(x, \Gamma) < \varepsilon\} \quad (4)$$

of a Δ -periodic net of surfaces separating the composite components. Distinguish a long-wave temperature field $\theta_L(z)$ which can be represented in a form:

$$\theta_L(z) = \begin{cases} \mathcal{G}(z) + h(x)\psi(z) & \text{for } z \in o_\varepsilon(\Gamma) \\ 0 & \text{for } z \notin o_\varepsilon(\Gamma) \end{cases} \quad (5)$$

differentiable for $z \in o_\varepsilon(\Gamma) \setminus \Gamma$ and for appropriately chosen right side of Eq. (4), for which a normal component of heat flux disappears on Γ :

$$(q_s)_n \equiv k(\nabla \theta_s)_n = 0 \quad (6)$$

Moreover, distinguish a short-wave temperature field θ_s which is differentiable any number of times and whose carrier $\text{supp}(\theta_s) = \{x \in R^D : \theta_s(x) \neq 0\}$ is localized beyond the ε -band $o_\varepsilon(\Gamma)$, i.e. $\text{supp}(\theta_s) \cap o_\varepsilon(\Gamma) = \emptyset$. The basis of these considerations is a possibility of a decomposition of the temperature field, proved in Kula (2015) and presented further.

Micro-macro-decomposition of temperature field

The temperature field θ in a periodic composite can be presented as a sum

$$\theta \equiv \theta_L + \theta_s \quad (7)$$

of a long-wave part θ_L and short-wave part θ_s of the temperature field. In Kula (2015) it was referred to as the LS-decomposition of temperature field.

The regularity of the field θ_s allows for a representation of this field in a form

$$\theta_s(y, z, t) \equiv \theta(y, z, t) - \theta_L(y, z, t) = \lambda a_p(z, t) \varphi^p(y, z) \quad (8)$$

as a development of the field θ_s in a Fourier series with respect to any appropriately chosen, orthogonal base $\varphi^p(y, z)$, independent on thermal and material properties of the composite. Moreover, in Eq. (8), the summation convention holds with respect to an integral positive superscript p . If one additionally assumes that the temperature $\langle \theta \rangle = \langle \theta \rangle(y, z)$ averaged in the field of repeatable cells $\Delta = \Delta(y)$ is independent on the variable $y \in R^d$, i.e. $\langle \theta \rangle = \langle \theta \rangle(z)$, then a limit transition $\varepsilon \rightarrow 0$ allows to derive model equations of heat conduction which are satisfied in Ω :

$$\begin{aligned} \langle c \rangle \dot{u} - \nabla^T (\langle k \rangle \nabla u + [k]^p a_p) &= -\langle b \rangle \\ \lambda^2 (\langle \varphi^p c \varphi^q \rangle \dot{a}_q - \nabla_z^T \langle \varphi^p c \varphi^q \rangle \nabla_z a_q) + \\ + 2\lambda s^{pq} \nabla_z a_q + \{k\}^{pq} a_p &= \langle \varphi^p b \rangle - ([k]^T)^p \nabla_z u \end{aligned} \quad (9)$$

for $z \in \Omega \setminus \Gamma$ and

$$\begin{aligned} \langle c \rangle \dot{u} - \nabla^T (\langle k \rangle \nabla u + \langle k \nabla^T h^A \rangle \psi_A) &= -\langle b \rangle \\ \langle \nabla_z^T h^A k \nabla_z h^B \rangle \psi_B + \langle \nabla^T h^A k \rangle \nabla_z u &= 0 \end{aligned} \quad (10)$$

For $z \in \Gamma$. In Eqs. (9) and (10) $A, B = 1, 2, \dots, N, p, q = 1, 2, \dots$, (Wodzyński, Kula & Wierzbicki, 2018), and the unknown functions are:

- a field of the average temperature $\langle \theta \rangle = \langle \theta \rangle(z)$
- a field of amplitudes of Fourier fluctuations, $a_p = a_p(z)$,
- a field of amplitudes of Woźniak fluctuations, $\Psi_p = \Psi_p(z)$.

In Eq. (9), following denotations have been assumed:

$$\begin{aligned} ([k]^T)^p &= \langle k \nabla^T \varphi^p \rangle, & [k]^p &= \langle k \nabla^T \varphi^p \rangle \\ 2s^{pq} &= \langle \nabla_y^T \varphi^p k \varphi^q \rangle - \langle \nabla^T \varphi^q k \varphi^p \rangle \\ \{k\}^{pq} &= \langle \nabla_y^T \varphi^p k \nabla \varphi^q \rangle \end{aligned} \quad (11)$$

Formulas in (10) are satisfied in Γ , thus they are not used in the analysis of the boundary effect.

The exact derivation we can find in the previous works of the authors (Kula, 2016; Wodzyński, Kula & Wierzbicki, 2018).

Composite behaviour of boundary effect

In the theory of ODE's, it is sometimes preferred a method of their solving by seeking solutions in a form of a sum of general integral of homogeneous equation and particular integral of non-homogeneous equation. The homogeneous part

$$\lambda^2 (\langle \varphi^p c \varphi^q \rangle \dot{a}_q - \nabla_z^T \langle \varphi^p c \varphi^q \rangle \nabla_z a_q) + 2\lambda s^{pq} \nabla_z a_q + \{k\}^{pq} a_p = 0 \quad (12)$$

of the second equation of the set (9) is exactly the same which is treated as a model of the composite behaviour of boundary effect serving as a description of transfer of boundary loads by the composite. In this work, they are thermal loads.

BOUNDARY EFFECT BEHAVIOUR EVOKED BY A TRIPLET OF FOURIER FLUCTUATIONS

Mathematical model of boundary effect behaviour

The Fourier base will be built in this work of:

- odd Fourier fluctuations (with an integral frequency ν_2)

$$f_2(\nu_2, y) = \begin{cases} \lambda \cos[(2\nu_2 - 1)\pi(\frac{y}{\lambda\eta_1} + 1)] & \text{for } -\lambda\eta_1 \leq y \leq 0 \\ \lambda \cos[(2\nu_2 - 1)\pi(\frac{y}{\lambda\eta_1} + 1)] & \text{for } 0 \leq y \leq \lambda\eta_1 \end{cases} \quad (13)$$

- even asymmetric Fourier fluctuations (with an integral frequency ν_1)

$$f_1(\nu_1, y) = \begin{cases} -\frac{\lambda}{2} [-1 + \cos 2\pi\nu_1(\frac{y}{\lambda\eta_1} + 1)] & \text{for } -\lambda\eta_1 \leq y \leq 0 \\ \frac{\lambda}{2} [-1 + \cos 2\pi\nu_1(\frac{y}{\lambda\eta_1} - 1)] & \text{for } 0 \leq y \leq \lambda\eta_1 \end{cases} \quad (14)$$

- even symmetric Fourier fluctuations (with an integral frequency ν_1)

$$f_3(v_1, y) = \begin{cases} \frac{\lambda}{2} \cos 2\pi v_1 \left(\frac{y}{\lambda \eta_I} + 1 \right) & \text{for } -\lambda \eta_I \leq y \leq 0 \\ \frac{\lambda}{2} \cos 2\pi v_1 \left(\frac{y}{\lambda \eta_{II}} - 1 \right) & \text{for } 0 \leq y \leq \lambda \eta_{II} \end{cases} \quad (15)$$

The names of the fluctuations – even or odd – are determined by the coefficients: even $2v_1$ and odd $2v_2 - 1$, respectively Eqs. (13), (14) and (15). If infinite column vectors of fluctuation amplitudes

$$f_1(v_1 = 1, y), f_1(v_1 = 2, y), \dots, f_2(v_2 = 1, y), f_1(v_2 = 2, y), \dots, f_3(v_1 = 1, y), f_1(v_1 = 2, y), \dots$$

are denoted by

$$b_1 = b_1(z) = [b_1^{(1)}(z), b_1^{(2)}(z), \dots]^T, a_1 = a_2(z) = [a_2^{(1)}(z), a_2^{(2)}(z), \dots]^T, b_3 = b_3(z) = [b_3^{(1)}(z), b_3^{(2)}(z), \dots]^T,$$

respectively, then Eq. (12) of the boundary effect behaviour can be rewritten in a form

$$\begin{aligned} \lambda^2 A_{11} b_1 - \lambda \beta_{12} a_2 - \{A\}_{11} b_1 &= 0 \\ \lambda^2 A_{22} a_2 - \lambda \beta_{21} b_1 - \beta_{23} b_3 - \{A\}_{22} a_2 &= 0 \\ \lambda^2 A_{33} b_3 - \lambda \beta_{32} a_2 - \{A\}_{33} b_3 &= 0 \end{aligned} \quad (16)$$

In the above, new form (16) of the boundary effect behaviour model, the coefficients are square diagonal matrices of infinite dimension:

- A_{11} is the diagonal matrix with the diagonal filled by coefficients of a form $\langle k f_1(\mathbf{n}_1, y) f_1(\mathbf{n}_1, y) \rangle$ in the order determined by v_1 ,
- A_{33} is the diagonal matrix with the diagonal filled by coefficients of a form $\langle k f_3(\mathbf{n}_1, y) f_3(\mathbf{n}_1, y) \rangle$ in the order determined by v_1 ,
- A_{22} is the diagonal matrix with the diagonal filled by coefficients of a form $\langle k f_2(\mathbf{n}_2, y) f_2(\mathbf{n}_2, y) \rangle$ in the order determined by v_2 ,
- $\{A\}_{11}$ is the diagonal matrix with the diagonal filled by coefficients of a form $\langle k \nabla^T f_1(\mathbf{n}_1, y) \nabla f_1(\mathbf{n}_1, y) \rangle$ in the order determined by v_1 ,
- $\{A\}_{33}$ is the diagonal matrix with the diagonal filled by coefficients of a form $\langle k \nabla^T f_3(\mathbf{n}_1, y) \nabla f_3(\mathbf{n}_1, y) \rangle$ in the order determined by v_1 ,
- $\{A\}_{22}$ is the diagonal matrix with the diagonal filled by coefficients of a form $\langle k \nabla^T f_2(\mathbf{n}_2, y) \nabla f_2(\mathbf{n}_2, y) \rangle$ in the order determined by v_2 ,

as well as square non-diagonal matrices of infinite dimension

$$\begin{aligned} -\beta_{12} &= -(\beta_{21})^T = \{ \langle k f_1(v_1, y) \nabla f_2(v_2, y) \rangle - \langle k \nabla^T f_1(v_1, y) \nabla f_2(v_2, y) \rangle \}_{1 \leq v_1 < +\infty, 1 \leq v_2 < +\infty} \\ -\beta_{32} &= -(\beta_{23})^T = \{ \langle k f_3(v_1, y) \nabla f_2(v_2, y) \rangle - \langle k \nabla^T f_3(v_1, y) \nabla f_2(v_2, y) \rangle \}_{1 \leq v_1 < +\infty, 1 \leq v_2 < +\infty} \end{aligned}$$

where the rows are numbered by the subscript v_1 and the columns by the subscript v_2 .

It must be emphasized that the coefficients of the set of the boundary effect equations present the below symmetries:

$$\begin{aligned} \beta_{12} &= -\beta_{21}, \quad \beta_{32} = -\beta_{23}, \quad \beta_{13} = -\beta_{31} \\ A_{11} &= A_{33}, \quad \{A\}_{11} = \{A\}_{33} = \text{diag} \left\{ \left(\frac{2v-1}{v} \right)^2 \right\} \cdot \{A\}_{22} \end{aligned} \quad (17)$$

where $\text{diag} \left\{ \left(\frac{2v-1}{v} \right)^2 \right\} = \text{diag} \left\{ \left(\frac{2 \cdot 1 - 1}{1} \right)^2, \left(\frac{2 \cdot 2 - 1}{2} \right)^2, \left(\frac{2 \cdot 3 - 1}{3} \right)^2, \dots \right\}$ is an infinity diagonal matrix with a diagonal filled by the numbers $\left(\frac{2v-1}{v} \right)^2$ in the order determined by $1 < v < +\infty$.

Boundary effect evoked by triplet of Fourier fluctuations

If one denotes:

$$\begin{aligned} \alpha &= A_{11} = A_{33}, & \alpha_2 &= A_{22} \\ \alpha_2 \gamma_2 &= \{A\}_{22}, & \alpha \gamma &= \{A\}_{11} = \{A\}_{33} \end{aligned} \quad (18)$$

then the equation set of the boundary effect evoked by triplet of Fourier fluctuations – the odd with the amplitude b_3 , the asymmetric even with the amplitude a_2 and the symmetric even with the amplitude a_3 – can be written as a set of three ODE's:

$$\begin{aligned} I: & \lambda^2 \alpha b_1'' - \alpha \gamma b_1' = \lambda \beta_{12} a_2' \\ II: & \lambda^2 \alpha_2 a_2'' - \alpha_2 \gamma_2 a_2' = \lambda (\beta_{21} b_1 + \beta_{23} b_3)' \\ III: & \lambda^2 \alpha b_3'' - \alpha \gamma b_3' = \lambda \beta_{32} a_2' \end{aligned} \quad (19)$$

This set contains the differential equations of the second order and has very interesting mathematical properties determined by its construction. The first and third equations contain components with a first derivative related exclusively to the amplitude a_2 of the even fluctuation (shortly – even amplitude), whereas it does not contain components with a first derivative related to the amplitudes of the odd fluctuation b_1 and b_3 (shortly – odd amplitudes). From the point of view of the simplest method of solving such set – the method of substitution – it means a possibility of reduction of this set to:

- 1° a single 4th order ODE for the even amplitude a_2 , and two 2nd order ODE's: for the odd amplitude b_1 and for the odd amplitude b_3 , both of them controlled by the even amplitude a_2 ;
- 2° two single 8th order ODE's for each of the odd amplitudes b_1 and b_3 , and one 2nd order ODE for the even amplitude a_2 , controlled by the odd amplitudes b_1 and b_3 .

Procedure 1°: The second equation in the set (19) can be reduced (without its differentiation and with use of the information contained in the remaining two equations) to a 4th order equation for the amplitude a_2 :

$$\begin{aligned} \beta_{21} I + \beta_{23} III: & \lambda^2 \alpha (\beta_{21} b_1'' + \beta_{23} b_3'') - \lambda (\beta_{21} \beta_{12} + \beta_{23} \beta_{32}) a_2' - \alpha \gamma (\beta_{21} b_1 + \beta_{23} b_3) = 0 \\ & \lambda (\beta_{21} \beta_{12} + \beta_{23} \beta_{32}) a_2' = \lambda^2 \alpha (\beta_{21} b_1 + \beta_{23} b_3)'' - \alpha \gamma (\beta_{21} b_1 + \beta_{23} b_3) \\ II \mapsto \beta_{21} I + \beta_{23} III: & \lambda (\beta_{21} \beta_{12} + \beta_{23} \beta_{32}) a_2' = \lambda \alpha (\lambda^2 \alpha_2 a_2'' - \alpha_2 \gamma_2 a_2') - \alpha \gamma (\beta_{21} b_1 + \beta_{23} b_3) \\ & \beta_{21} b_1 + \beta_{23} b_3 = \lambda \gamma^{-1} (\lambda^2 \alpha_2 a_2'' - \alpha_2 \gamma_2 a_2') - \lambda \alpha^{-1} \gamma^{-1} (\beta_{21} \beta_{12} + \beta_{23} \beta_{32}) a_2' \\ (II \mapsto \beta_{21} I + \beta_{23} III) \mapsto & \\ \mapsto II: & \lambda [\lambda \gamma^{-1} (\lambda^2 \alpha_2 a_2'' - \alpha_2 \gamma_2 a_2') - \lambda \alpha^{-1} \gamma^{-1} (\beta_{21} \beta_{12} + \beta_{23} \beta_{32}) a_2']' = \lambda^2 \alpha_2 a_2'' - \alpha_2 \gamma_2 a_2' \\ & \lambda^4 \alpha_2 a_2'''' - \lambda^2 \alpha_2 \gamma_2 a_2'' - \lambda^2 \alpha^{-1} (\beta_{21} \beta_{12} + \beta_{23} \beta_{32}) a_2'' = \lambda^2 \alpha_2 \gamma_2 a_2'' - \alpha_2 \gamma_2 \gamma a_2' \\ & \lambda^4 \alpha_2 a_2'''' - \lambda^2 [\alpha^{-1} (\beta_{21} \beta_{12} + \beta_{23} \beta_{32}) + \alpha_2 \gamma_2 + \alpha_2 \gamma] a_2'' + \alpha_2 \gamma \gamma_2 a_2' = 0 \\ III: & \lambda^2 \alpha b_1'' - \lambda \beta_{12} a_2' - \alpha \gamma b_1' = 0 \\ II: & \lambda^2 \alpha_2 a_2'' - \lambda \beta_{21} b_1' - \lambda \beta_{23} b_3' - \alpha_2 \gamma_2 a_2' = 0, \quad \lambda (\beta_{21} b_1 + \beta_{23} b_3)' = \lambda^2 \alpha_2 a_2'' - \alpha_2 \gamma_2 a_2' \\ III: & \lambda^2 \alpha b_3'' - \lambda \beta_{32} a_2' - \alpha \gamma b_3' = 0 \end{aligned} \quad (20)$$

Hence, the set (19) with the transformed second equation and unchanged first and third equations assumes a form:

$$\begin{cases}
 I: \lambda^2 \alpha b_1'' - \lambda \beta_{12} a_2' - \alpha \gamma b_1 = 0 \\
 III: \lambda^2 \alpha b_3'' - \lambda \beta_{32} a_2' - \alpha \gamma b_3 = 0
 \end{cases}$$

$$(II \mapsto \beta_{21} I + \beta_{23} III) \mapsto$$

$$\begin{aligned}
 \mapsto II: \lambda^4 \alpha_2 a_2'''' - \lambda^2 [\alpha^{-1} (\beta_{21} \beta_{12} + \beta_{23} \beta_{32}) + \alpha_2 \gamma_2 + \alpha_2 \gamma] a_2'' + \alpha_2 \gamma \gamma_2 a_2 = 0 \\
 \lambda^4 \alpha_2 a_2'''' - \lambda^2 [\alpha^{-1} (\beta_{21} \beta_{12} + \beta_{23} \beta_{32}) + \alpha_2 \gamma_2 + \alpha_2 \gamma] a_2'' + \alpha_2 \gamma \gamma_2 a_2 = 0 \\
 \lambda^4 a_2'''' - \lambda^2 (\gamma_2 + \gamma - \alpha_2^{-1} \alpha^{-1} \beta^2) a_2'' + \gamma_2 \gamma a_2 = 0
 \end{aligned}
 \tag{21}$$

Procedure 2°: The subset consisting of the first and third equations in the set (19) (like the second equation in the Procedure 1°) can be replaced (without their double side differentiation and with use of the information contained in the remaining second equation) by a set of equations for the odd amplitudes b_1 and b_3 , independent on the even amplitude a_2 . Further are presented subsequent stages of the discussed procedure.

Stage 1:

$$\begin{aligned}
 I: \lambda^2 \alpha b_1'' - \alpha \gamma b_1 &= \lambda \beta_{12} a_2' \\
 \beta_{12} II': \lambda^2 \alpha_2 \beta_{12} a_2'' - \lambda \beta_{12} \beta_{21} b_1'' - \lambda \beta_{12} \beta_{23} b_3'' - \gamma_2 \alpha_2 \beta_{12} a_2' &= 0 \\
 II: \lambda^2 \alpha_2 a_2'' - \lambda \beta_{21} b_1'' - \lambda \beta_{23} b_3'' - \alpha_2 \gamma_2 a_2 &= 0 \\
 III: \lambda^2 b_3'' - \alpha \gamma b_3 &= \lambda \beta_{32} a_2' \\
 \beta_{32} III': \lambda^2 \alpha_2 \beta_{32} a_2'' - \lambda \beta_{32} \beta_{21} b_1'' - \lambda \beta_{32} \beta_{23} b_3'' - \gamma_2 \alpha_2 \beta_{32} a_2' &= 0
 \end{aligned}
 \tag{22}$$

Stage 2:

$$\begin{aligned}
 I: \lambda^2 \alpha b_1'' - \alpha \gamma b_1 &= \lambda \beta_{12} a_2' \\
 Ia (I \rightarrow II): \gamma_2 a_2 &= \lambda \alpha_2 (\lambda^2 \alpha b_1'' - \alpha \gamma b_1)' - \lambda \beta_{12} \beta_{21} b_1' - \lambda \beta_{12} \beta_{23} b_3' \\
 \gamma_2 a_2 &= \lambda^3 \alpha_2 \alpha b_1''' - \lambda (\alpha_2 \alpha \gamma + \beta_{12} \beta_{21}) b_1' - \lambda \beta_{12} \beta_{23} b_3' \\
 II: \lambda^2 \alpha_2 a_2'' - \lambda \beta_{21} b_1'' - \lambda \beta_{23} b_3'' - \alpha_2 \alpha a_2 &= 0 \\
 IIIa (III \rightarrow II): \gamma_2 a_2 &= \lambda \alpha_2 (\lambda^2 b_3'' - \alpha \gamma b_3)' - \lambda \beta_{32} \beta_{21} b_1' - \lambda \beta_{32} \beta_{23} b_3' \\
 \gamma_2 a_2 &= \lambda^3 \alpha_2 \alpha b_3''' - \lambda (\alpha_2 \alpha \gamma + \beta_{32} \beta_{23}) b_3' - \lambda \beta_{32} \beta_{21} b_1' \\
 III: \lambda^2 \alpha b_3'' - \alpha \gamma b_3 &= \lambda \beta_{32} a_2' \lambda^4 a_2'''' - \lambda^2 (\gamma_2 + \gamma - \alpha_2^{-1} \alpha^{-1} \beta^2) a_2'' + \gamma_2 \gamma a_2 = 0
 \end{aligned}
 \tag{23}$$

Stage 3:

$$\begin{aligned}
 Ia: \gamma_2 a_2 &= \lambda \alpha_2 (\lambda^2 \alpha b_1'' - \alpha \gamma b_1)' - \lambda \beta_{12} \beta_{21} b_1' - \lambda \beta_{12} \beta_{23} b_3' \\
 \gamma_2 a_2 &= \lambda^3 \alpha_2 \alpha b_1''' - \lambda (\alpha_2 \alpha \gamma + \beta_{12} \beta_{21}) b_1' - \lambda \beta_{12} \beta_{23} b_3' \\
 Ib (Ia \rightarrow \gamma_2 I): \lambda^2 \alpha \gamma_2 b_1'' - \alpha \gamma \gamma_2 b_1 &= \lambda \beta_{12} \gamma_2 a_2' = \lambda \beta_{12} [\lambda^3 \alpha_2 \alpha b_3''' - \lambda (\alpha_2 \alpha \gamma + \beta_{32} \beta_{23}) b_3' - \lambda \beta_{32} \beta_{21} b_1'] \\
 \lambda^4 \alpha_2 \beta_{12} b_1'''' - \lambda^2 [\beta_{12} (\alpha_2 \gamma + \alpha^{-1} \beta_{12} \beta_{21}) + \gamma_2] b_1'' &+ \gamma \gamma_2 b_1 - \lambda^2 \alpha^{-1} \beta_{12} \beta_{12} \beta_{23} b_3'' = 0
 \end{aligned}$$

$$\begin{aligned}
 II : \lambda^2 \alpha_2 a_2'' - \lambda \beta_{21} b_1' - \lambda \beta_{23} b_3' - \alpha_2 \alpha a_2 &= 0 \\
 IIIb \text{ (IIIa} \rightarrow \gamma_2 \text{III)} : \lambda^2 \alpha \gamma_2 b_3'' - \alpha \gamma \gamma_2 b_3' &= \lambda \beta_{32} \gamma_2 a_2' = \lambda \beta_{32} \gamma_2 [\lambda^3 \alpha_2 \alpha b_3'''' - \lambda (\alpha_2 \alpha \gamma + \beta_{32} \beta_{23}) b_3' - \lambda \beta_{32} \beta_{21} b_1'] \\
 &\quad \lambda^4 \alpha_2 \beta_{32} b_3'''' - \lambda^2 [\beta_{32} (\alpha_2 \gamma + \alpha^{-1} \beta_{32} \beta_{23}) + \gamma_2] b_3'' + \gamma \gamma_2 b_3' - \lambda^2 \alpha^{-1} \beta_{32} \beta_{32} \beta_{21} b_1'' = 0 \\
 IIIa : \gamma_2 a_2 = \lambda \alpha_2 (\lambda^2 b_3'' - \alpha \gamma b_3') - \lambda \beta_{32} \beta_{21} b_1' - \lambda \beta_{32} \beta_{23} b_3' \\
 \gamma_2 a_2 = \lambda^3 \alpha_2 \alpha b_3'''' - \lambda (\alpha_2 \alpha \gamma + \beta_{32} \beta_{23}) b_3' - \lambda \beta_{32} \beta_{21} b_1' & \quad (24)
 \end{aligned}$$

Thus, the set (19) with the transformed first and third equations and unchanged second one assumes a form:

$$\begin{cases}
 \lambda^4 \alpha_2 \beta_{12} b_1'''' - \lambda^2 [\beta_{12} (\alpha_2 \gamma + \alpha^{-1} \beta_{12} \beta_{21}) + \gamma_2] b_1'' + \gamma \gamma_2 b_1' - \lambda^2 \alpha^{-1} \beta_{12} \beta_{12} \beta_{23} b_3'' = 0 \\
 \lambda^4 \alpha_2 \beta_{32} b_3'''' - \lambda^2 [\beta_{32} (\alpha_2 \gamma + \alpha^{-1} \beta_{32} \beta_{23}) + \gamma_2] b_3'' + \gamma \gamma_2 b_3' - \lambda^2 \alpha^{-1} \beta_{32} \beta_{32} \beta_{21} b_1'' = 0 \\
 \lambda^2 \alpha_2 a_2'' - \lambda (\beta_{21} b_1' + \beta_{23} b_3') - \alpha_2 \alpha a_2 = 0
 \end{cases} \quad (25)$$

or simpler

$$\begin{cases}
 (i) : \lambda^4 (\beta_{23} b_3)'''' - \lambda^2 [\gamma + \alpha_2^{-1} \alpha^{-1} \beta_{32} \beta_{23} + \gamma_2 \beta_{32}^{-1}] (\beta_{23} b_3)'' + \gamma \gamma_2 \alpha_2^{-1} \beta_{32}^{-1} (\beta_{23} b_3)' = \lambda^2 \alpha_2^{-1} \alpha^{-1} \beta_{23} \beta_{32} (\beta_{21} b_1)'' \\
 (ii) : \lambda^4 (\beta_{21} b_1)'''' - \lambda^2 [\gamma + \alpha_2^{-1} \alpha^{-1} \beta_{12} \beta_{21} + \gamma_2 \beta_{12}^{-1}] (\beta_{21} b_1)'' + \gamma \gamma_2 \alpha_2^{-1} \beta_{12}^{-1} (\beta_{21} b_1)' = \lambda^2 \alpha_2^{-1} \alpha^{-1} \beta_{21} \beta_{12} (\beta_{23} b_3)'' \\
 (iii) : \lambda^2 a_2'' - \alpha a_2 = \lambda \alpha_2^{-1} (\beta_{21} b_1' + \beta_{23} b_3)'
 \end{cases} \quad (26)$$

One of the equations in the set (26) for the odd amplitudes b_1 and b_3 – here the first one has been chosen – can be changed into a single 6th order equation, independent on the second of these two equations, for the amplitude b_1 . A double side differentiation of this chosen equation is not necessary and the second of these two equations can be use in this purpose:

$$\begin{aligned}
 (ii) \mapsto \alpha_2^{-1} \alpha^{-1} \beta_{21} \beta_{12} \cdot (i) : \\
 \lambda^4 \{ \lambda^4 (\beta_{21} b_1)'''' - \lambda^2 [\gamma + \alpha_2^{-1} \alpha^{-1} \beta_{12} \beta_{21} + \gamma_2 \beta_{12}^{-1}] (\beta_{21} b_1)'' + \gamma \gamma_2 \alpha_2^{-1} \beta_{12}^{-1} (\beta_{21} b_1)' \}'' - \\
 - \lambda^2 [\gamma + \alpha_2^{-1} \alpha^{-1} \beta_{32} \beta_{23} + \gamma_2 \beta_{32}^{-1}] \cdot \{ \lambda^4 (\beta_{21} b_1)'''' - \lambda^2 [\gamma + \alpha_2^{-1} \alpha^{-1} \beta_{12} \beta_{21} + \gamma_2 \beta_{12}^{-1}] \times \\
 \times (\beta_{21} b_1)'' + \gamma \gamma_2 \alpha_2^{-1} \beta_{12}^{-1} (\beta_{21} b_1)' \} + \gamma \gamma_2 \alpha_2^{-1} \beta_{32}^{-1} \{ \lambda^4 (\beta_{21} b_1)'''' - \lambda^2 [\gamma + \alpha_2^{-1} \alpha^{-1} \beta_{12} \beta_{21} + \gamma_2 \beta_{12}^{-1}] \times \\
 \times (\beta_{21} b_1)'' + \gamma \gamma_2 \alpha_2^{-1} \beta_{12}^{-1} (\beta_{21} b_1)' \} = \lambda^2 \alpha_2^{-1} \alpha^{-1} \beta_{21} \beta_{12} \alpha_2^{-1} \alpha^{-1} \beta_{23} \beta_{32} (\beta_{21} b_1)'' \\
 (ii) : \lambda^4 (\beta_{21} b_1)'''' - \lambda^2 [\gamma + \alpha_2^{-1} \alpha^{-1} \beta_{12} \beta_{21} + \gamma_2 \beta_{12}^{-1}] (\beta_{21} b_1)'' + \gamma \gamma_2 \alpha_2^{-1} \beta_{12}^{-1} (\beta_{21} b_1)' = \lambda^2 \alpha_2^{-1} \alpha^{-1} \beta_{21} \beta_{12} (\beta_{23} b_3)'' \\
 (iii) : \lambda^2 a_2'' - \alpha a_2 = \lambda \alpha_2^{-1} (\beta_{21} b_1' + \beta_{23} b_3)'
 \end{aligned} \quad (27)$$

The first of these equations, denoted as $(ii) \mapsto \alpha_2^{-1} \alpha^{-1} \beta_{21} \beta_{12} \cdot (i)$, will not be analysed in this work, hence it will be no longer simplified. Nevertheless, it must be emphasized that this equation is proportional to λ^2 – it can be divided by λ^2 .

The sets (21), (26) and (27) are a base for controlling the boundary effect with use of boundary conditions.

Special cases. Controlling boundary conditions

Boundary effect evoked by a doublet of Fourier fluctuations and a single Fourier fluctuation

Including in the set (27) the boundary conditions resulting in the disappearance of the amplitude b_1 over the whole partition width, e.g.

$$b_1(z=0) = b_1(z=\delta) = 0, \quad \frac{db_1(z=0)}{dz} = \frac{db_1(z=\delta)}{dz} = 0, \quad \frac{d^2b_1(z=0)}{dz^2} = \frac{d^2b_1(z=\delta)}{dz^2} = 0 \quad (28)$$

one obtains a reduced form of the set (27):

$$\begin{aligned}
 (ii) &\mapsto \alpha_2^{-1} \alpha^{-1} \beta_{21} \beta_{12} \cdot (i): \quad b_1 = 0 \\
 (ii): \quad &\alpha_2^{-1} \alpha^{-1} \beta_{21} \beta_{12} (\beta_{23} b_3)'' = 0, \quad b_3(z) = z \cdot \frac{db_3(z=0)}{dz} + b_3(z=0) \\
 (iii): \quad &a_2'' - \alpha a_2 = \alpha_2^{-1} \beta_{23} \cdot \frac{db_3(z=0)}{dz}
 \end{aligned} \tag{29}$$

describing the boundary effect behaviour where the odd fluctuation with the amplitude b_1 is not transferred across the partition and the transfer of the odd fluctuation with the amplitude b_3 cooperating with the even fluctuation with the amplitude b_2 is described by the set (29).

Boundary effect evoked by a single Fourier fluctuation

Including in the set (27) the boundary conditions resulting in the disappearance of the amplitude a_2 over the whole partition width, e.g.

$$a_2(z=0) = a_2(z=\delta) = 0, \quad \frac{da_2(z=0)}{dz} = \frac{da_2(z=\delta)}{dz} = 0 \tag{30}$$

one obtains a reduced form of the set (27):

$$\begin{cases}
 I: \quad \alpha b_1'' - \alpha \gamma b_1 = 0 \\
 III: \quad \alpha b_3'' - \alpha \gamma b_3 = 0
 \end{cases} \tag{31}$$

$$(II \mapsto \beta_{21} I + \beta_{23} III) \mapsto II: \quad a_2 = 0$$

describing the boundary effect behaviour where the even fluctuation with the amplitude a_2 is not transferred across the partition and the independent transfer of the odd fluctuations with the amplitudes b_1 and b_3 is described by the set (20).

Solution of the set (24), assuming the values $b_1(z=0)$ and $b_1(z=\delta)$ at the external and internal sides of the partition, can be written in the form:

$$b_1(z) = - \frac{\sinh \sqrt{\gamma} \cdot \frac{z-\delta}{\lambda}}{\sinh \sqrt{\gamma} \cdot \frac{\delta}{\lambda}} \cdot b_1(z=0) + \frac{\sinh \sqrt{\gamma} \cdot \frac{z}{\lambda}}{\sinh \sqrt{\gamma} \cdot \frac{\delta}{\lambda}} \cdot b_1(z=\delta) \tag{32}$$

(cf. Kula, 2016).

ANALYSIS

Reduction of the model form to a 4th order ODE's

The subject of the analysis in this section is a single 4th order ODE

$$a_2'''' - \lambda^{-2} [\gamma_2 + \gamma + \alpha_2^{-1} \alpha^{-1} (\beta_{21} \beta_{12} + \beta_{23} \beta_{32})] a_2'' + \lambda^{-4} \gamma_2 \gamma a_2 = 0 \tag{33}$$

obtained from the last, leading equation of the set (14) by double-sided division of this equation by λ^4 . Calculation of the even amplitude a_2 from Eq. (7) and the odd amplitudes b_1 and b_3 from Eq. (5) leads to the solution of the set (4). The exact derivation we can find in the previous works of the authors (Wodzyński, Kula & Wierzbicki, 2018).

It must be emphasized that the way of solving the problem (here: the method of substitution) results in a way of imposing conditions of uniqueness, in this case for a_2 , b_1 and b_3 . Boundary conditions for these amplitudes are formulated in the next subsection.

Boundary conditions (uniqueness conditions)

The boundary conditions for the even amplitude a_2 can be assumed in a form:

$$\begin{aligned} a_2(z)|_{z=0} &= a_0, & a_2(z)|_{z=\delta} &= a_\delta \\ \frac{da_2(z)}{dz}|_{z=0} &= a'_0, & \frac{da_2(z)}{dz}|_{z=\delta} &= a'_\delta \end{aligned} \quad (34)$$

and for the odd amplitudes b_1 and b_3 , taking (5) into consideration, in a form:

$$\begin{aligned} b_1(z)|_{z=0} &= b_{10}, & b_1(z)|_{z=\delta} &= b_{1\delta} \\ b_3(z)|_{z=0} &= b_{30}, & b_3(z)|_{z=\delta} &= b_{3\delta} \end{aligned} \quad (35)$$

Solution form

As $\beta_{21} = -\beta_{21}$ and $\beta_{23} = -\beta_{32}$, then $\beta_{21}\beta_{12} + \beta_{23}\beta_{32} = -(\beta_{21}^2\beta_{23}^2) < 0$ and finally $\beta_{21}\beta_{12} + \beta_{23}\beta_{32} = -\beta^2$ for some $\beta \neq 0$. Present Eq. (7) in a form:

$$\frac{d^4\omega(\xi)}{d^4\xi} - (\gamma_2 + \gamma - \alpha_2^{-1}\alpha^{-1}\beta^2) \frac{d^2\omega(\xi)}{d^2\xi} + \gamma_2\gamma\omega(\xi) = 0, \quad a_2(z) = \omega(\xi), \quad \xi = \lambda^{-1}z \quad (36)$$

free from the scale effect. The characteristic equation of this equation

$$r^4 - (\gamma_2 + \gamma - \alpha_2^{-1}\alpha^{-1}\beta^2)r^2 + \gamma_2\gamma a_2 = 0 \quad (37)$$

is a biquadratic equation generated by a quadratic equation

$$R^2 - (\gamma_2 + \gamma - \alpha_2^{-1}\alpha^{-1}\beta^2)R + \gamma_2\gamma = 0 \quad (38)$$

Considering values of the coefficients:

$$\begin{aligned} \beta^2 &= -(\beta_{21}\beta_{12} + \beta_{23}\beta_{32}) = \lambda^2 \cdot \left[\frac{(2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 \cdot \{ (k_1 + k_{II})^2 \cdot \nu_1^2 + (k_1 - k_{II})^2 \cdot (\nu_1 - \nu_2 + \frac{1}{2})^2 \} \\ \alpha_2 &= \left\{ \begin{array}{l} 0 \text{ for } \nu_1 \neq \nu_2 \\ \frac{\lambda^2}{2} (k_1\eta_1 + k_{II}\eta_{II}) = \frac{\lambda^2}{2} \langle k \rangle \text{ for } \nu_1 = \nu_2 = \nu \end{array} \right\}, \quad \alpha = \alpha(\mu) = \left\{ \begin{array}{l} \frac{\lambda^2}{4} (k_1\eta_1 + k_{II}\eta_{II}) \text{ for } \nu_1 \neq \nu_2 \\ \frac{3}{8} \lambda^2 (k_1\eta_1 + k_{II}\eta_{II}) \text{ for } \nu_1 = \nu_2 = \mu \end{array} \right\} \\ \alpha_2\gamma_2 &= \left\{ \begin{array}{l} 0 \text{ for } \nu_1 \neq \nu_2 \\ \frac{1}{2} (2\nu_1 - 1)(2\nu_2 - 1) \pi^2 \cdot \left(\frac{k_1}{\eta_1} + \frac{k_{II}}{\eta_{II}} \right) \text{ for } \nu_1 = \nu_2 \end{array} \right\}, \quad \gamma_2 = \gamma_2(\nu) = \left\{ \begin{array}{l} 0 \text{ for } \nu_1 \neq \nu_2 \\ (2\nu - 1)^2 \pi^2 \cdot \frac{\langle k \rangle_H}{\lambda^2 \langle k \rangle} \text{ for } \nu_1 = \nu_2 = \nu \end{array} \right\} \\ \alpha\gamma &= \frac{\pi^2 \nu_1 \nu_2}{2} \cdot \langle k \rangle_H \left\{ \begin{array}{l} 0 \text{ for } \nu_1 \neq \nu_2 \\ 1 \text{ for } \nu_1 = \nu_2 \end{array} \right\}, \quad \gamma = \gamma(\mu) = \left\{ \begin{array}{l} 0 \text{ for } \nu_1 \neq \nu_2 \\ \frac{1}{2} \mu^2 \pi^2 \cdot \left(\frac{k_1}{\eta_1} + \frac{k_{II}}{\eta_{II}} \right) \\ \frac{3\lambda^2}{8} \langle k \rangle \text{ for } \nu_1 = \nu_2 = \mu \end{array} \right\} = \frac{4}{3} \mu^2 \pi^2 \cdot \frac{\langle k \rangle_H}{\lambda^2 \langle k \rangle} \text{ for } \nu_1 = \nu_2 = \mu \end{aligned}$$

$$\begin{aligned}
 \gamma_2(\nu) + \gamma(\mu) &= (2\nu - 1)^2 \pi^2 \cdot \frac{\langle k \rangle_H}{\lambda^2 \langle k \rangle} + \frac{4}{3} \mu^2 \pi^2 \cdot \frac{\langle k \rangle_H}{\lambda^2 \langle k \rangle} = [(2\nu - 1)^2 + \frac{4}{3} \mu^2] \pi^2 \cdot \frac{\langle k \rangle_H}{\lambda^2 \langle k \rangle} \\
 \alpha_2^{-1} \alpha^{-1} \beta^2 &= \frac{\lambda^2 \cdot \left[\frac{(2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 \cdot \{ (k_1 + k_{II})^2 \cdot \nu_1^2 + (k_1 - k_{II})^2 \cdot (\nu_1 - \nu_2 + \frac{1}{2})^2 \}}{\frac{1}{2} \cdot \frac{3}{8} \cdot \lambda^2 \lambda^2 \langle k \rangle^2} = \\
 &= \frac{16}{3\lambda^2} \cdot \left[\frac{(2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 \cdot \frac{\{ (k_1 + k_{II})^2 \cdot \nu_1^2 + (k_1 - k_{II})^2 \cdot (\nu_1 - \nu_2 + \frac{1}{2})^2 \}}{\langle k \rangle^2} \\
 \gamma_2 + \gamma &= [(2\nu_2 - 1)^2 + \frac{4}{3} \nu_1^2] \pi^2 \cdot \frac{\langle k \rangle_H}{\lambda^2 \langle k \rangle} \\
 \gamma_2 \gamma &= \frac{4}{3} (2\nu_2 - 1)^2 \cdot \nu_1^2 \cdot \left[\pi^2 \cdot \frac{\langle k \rangle_H}{\lambda^2 \langle k \rangle} \right]^2 \\
 \gamma_2 + \gamma - \alpha_2^{-1} \alpha^{-1} \beta^2 &= [(2\nu_2 - 1)^2 + \frac{4}{3} \nu_1^2] \pi^2 \cdot \frac{\langle k \rangle_H}{\lambda^2 \langle k \rangle} + \frac{16}{3\lambda^2} \cdot \left[\frac{(2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 \times \\
 &\times \frac{\{ (k_1 + k_{II})^2 \cdot \nu_1^2 + (k_1 - k_{II})^2 \cdot (\nu_1 - \nu_2 + \frac{1}{2})^2 \}}{\langle k \rangle^2}
 \end{aligned} \tag{39}$$

as well as that

$$\begin{aligned}
 \Delta &= \frac{1}{4} (\gamma_2 + \gamma - \alpha_2^{-1} \alpha^{-1} \beta^2)^2 - \gamma_2 \gamma = \\
 &= \left\{ \left[(2\nu_2 - 1)^2 + \frac{4}{3} \nu_1^2 \right] \pi^2 \cdot \frac{\langle k \rangle_H}{\lambda^2 \langle k \rangle} + \frac{16}{3\lambda^2} \cdot \left[\frac{(2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 \cdot \frac{\{ (k_1 + k_{II})^2 \cdot \nu_1^2 + (k_1 - k_{II})^2 \cdot (\nu_1 - \nu_2 + \frac{1}{2})^2 \}}{\langle k \rangle^2} \right\}^2 - \\
 &- \frac{16}{3} (2\nu_2 - 1)^2 \cdot \nu_1^2 \cdot \left[\pi^2 \cdot \frac{\langle k \rangle_H}{\lambda^2 \langle k \rangle} \right]^2 = \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} \left\{ \left[(2\nu_1)^2 + (2\nu_2 - 1)^2 \right] - 2 \frac{(k_1 \alpha_1 + k_{II} \alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1 (2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 \right\}^2 - \\
 &- \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} [2\nu_1 (2\nu_2 - 1)]^2 = \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} \cdot \left\{ \left[(2\nu_1)^2 + (2\nu_2 - 1)^2 \right] - 2 \frac{(k_1 \alpha_1 + k_{II} \alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1 (2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 + 2\nu_1 (2\nu_2 - 1) \right\} \times \\
 &\times \left\{ \left[(2\nu_1)^2 + (2\nu_2 - 1)^2 \right] - 2 \frac{(k_1 \alpha_1 + k_{II} \alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1 (2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 - 2\nu_1 (2\nu_2 - 1) \right\}
 \end{aligned} \tag{40}$$

and assuming that

$$B = -\frac{1}{2} (\gamma_2 + \gamma - \alpha_2^{-1} \alpha^{-1} \beta^2), \quad C = \gamma_2 \gamma \tag{41}$$

one can rewrite the characteristic equation (37) in a more concise form:

$$r^4 + 2Br^2 + C = 0 \tag{42}$$

and the quadratic equation (38) in a form:

$$R^2 + 2BR + C = 0 \quad (43)$$

Such notation allows to clearly discuss forms of the solution of the 4th order ODE for the even amplitude $a_2 = a_2(z)$. Thus, it will be analysed a possibility of excitation of oscillations of amplitudes of the temperature fluctuations within the whole building partition forced to transport these fluctuations across the partition.

The coefficients (41) used in Eqs. (42) and (43) are the functions of:

- the frequency ν_1 of the even fluctuation with the amplitude a_2 and the frequency ν_2 of the odd fluctuations with the amplitudes b_1 and b_3 ,
- the saturation $0 < \eta_I < 1$ and saturation $0 < \eta_{II} = 1 - \eta_I < 1$ (determined with use of the saturation η_I) – they are geometrical fractions of the areas occupied by the components with conductivities k_I and k_{II} ,
- a quotient of the conductivities k_I and k_{II} .

The aforementioned analysis, being the subject of this work, consists in presentation of a form of the 4th order ODE (36) for the even amplitude $a_2(z)$.

When does a boundary thermal load cause a thermal pulsation of a composite partition?

1) Analysis for the frequencies $\nu_1 = 5$ and $\nu_2 = 6$

- **Numerical analysis of the sign of the determinant of the quadratic equation (38) being the characteristic equation of the equation for the amplitude $a_2(z)$ of the even fluctuation transported through a composite building partition and cooperating with two remaining Fourier fluctuations**

It has been found that the sign of the determinant Δ

$$\begin{aligned} \Delta &= \frac{1}{4}(\gamma_2 + \gamma - \alpha_2^{-1}\alpha^{-1}\beta^2)^2 - \gamma_2\gamma = \\ &= \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} \{ [(2\nu_1)^2 + (2\nu_2 - 1)^2] - 2 \frac{(k_I\alpha_1 + k_{II}\alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1(2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 \} - \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} [2\nu_1(2\nu_2 - 1)]^2 = \\ &= \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} \cdot \{ [(2\nu_1)^2 + (2\nu_2 - 1)^2] - 2 \frac{(k_I\alpha_1 + k_{II}\alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1(2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 + 2\nu_1(2\nu_2 - 1) \} \times \\ &\quad \times \{ [(2\nu_1)^2 + (2\nu_2 - 1)^2] - 2 \frac{(k_I\alpha_1 + k_{II}\alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1(2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 - 2\nu_1(2\nu_2 - 1) \} \end{aligned} \quad (44)$$

is always positive, negative or zero depending on the frequency ν_1 and ν_2 , saturations η_I , η_{II} and quotient of the conductivities k_{II}/k_I (cf. Fig. 1)

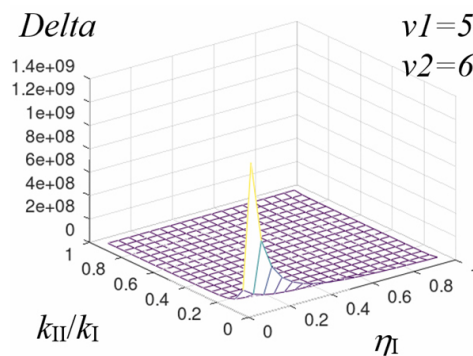


Fig. 1. Graph of the parameter Δ for first analysis

– **Numerical analysis of the roots of the quadratic equation (38) being the characteristic equation of the equation for the amplitude $a_2(z)$ of the even fluctuation transported through a composite building partition and cooperating with two remaining Fourier fluctuations**

The values of the determinant Δ and both roots of the quadratic equation (38) has been analysed as functions of the frequencies ν_1 and ν_2 , saturations η_I , η_{II} and quotient of the conductivities k_{II}/k_I (cf. Fig. 2)

$$\begin{aligned} \Delta &= \frac{1}{4}(\gamma_2 + \gamma - \alpha_2^{-1}\alpha^{-1}\beta^2)^2 - \gamma_2\gamma = \\ &= \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} \{ [(2\nu_1)^2 + (2\nu_2 - 1)^2] - 2 \frac{(k_I\alpha_1 + k_{II}\alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1(2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 \}^2 - \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} [2\nu_1(2\nu_2 - 1)]^2 = \\ &= \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} \cdot \{ [(2\nu_1)^2 + (2\nu_2 - 1)^2] - 2 \frac{(k_I\alpha_1 + k_{II}\alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1(2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 + 2\nu_1(2\nu_2 - 1) \} \times \\ &\quad \times \{ [(2\nu_1)^2 + (2\nu_2 - 1)^2] - 2 \frac{(k_I\alpha_1 + k_{II}\alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1(2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 - 2\nu_1(2\nu_2 - 1) \} \end{aligned} \quad (45)$$

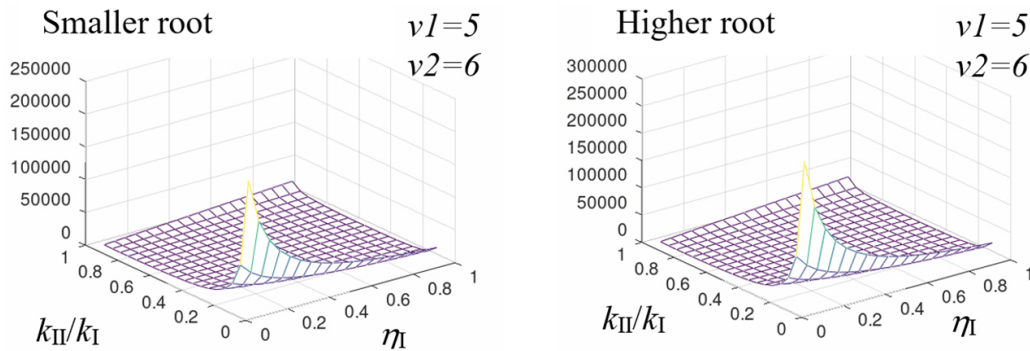


Fig. 2. Graphs of the smaller and higher root for first analysis

For the frequencies $\nu_1 = 5$ and $\nu_2 = 6$ being analysed, the determinant Δ and the both roots of the quadratic equation

$$\Delta > 0, \quad R_1 = B - \sqrt{\Delta} > 0, \quad R_2 = B + \sqrt{\Delta} > 0 \quad (46)$$

are positive. It means that the biquadratic equation (42) has four roots, two positive and two negative:

$$\Delta > 0, \quad R_{1+} = \sqrt{B - \sqrt{\Delta}} > 0, \quad R_{1-} = -\sqrt{B - \sqrt{\Delta}} < 0, \quad R_{2+} = \sqrt{B + \sqrt{\Delta}} > 0, \quad R_{2-} = -\sqrt{B + \sqrt{\Delta}} < 0 \quad (47)$$

Thus, the general integral of the ODE for the amplitude $a_2(z)$ has a form

$$a_2(z) = \omega\left(\frac{z}{\lambda}\right) = C_{1+} e^{R_{1+} \frac{z}{\lambda}} + C_{1-} e^{R_{1-} \frac{z}{\lambda}} + C_{2+} e^{R_{2+} \frac{z}{\lambda}} + C_{2-} e^{R_{2-} \frac{z}{\lambda}} \quad (48)$$

for any real constants C_{1+} , C_{1-} , C_{2+} , C_{2-} , possible to be determined from the boundary values (34).

2) Analysis for the frequencies $\nu_1 = 1$ and $\nu_2 = 1$

- **Numerical analysis of the sign of the determinant of the quadratic equation (38) being the characteristic equation of the equation for the amplitude $a_2(z)$ of the even fluctuation transported through a composite building partition and cooperating with two remaining Fourier fluctuations**

It has been found that the sign of the determinant Δ

$$\begin{aligned} \Delta &= \frac{1}{4}(\gamma_2 + \gamma - \alpha_2^{-1} \alpha^{-1} \beta^2)^2 - \gamma_2 \gamma = \\ &= \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} \{ [(2\nu_1)^2 + (2\nu_2 - 1)^2] - 2 \frac{(k_I \alpha_1 + k_{II} \alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1(2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 \}^2 - \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} [2\nu_1(2\nu_2 - 1)]^2 = \\ &= \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} \cdot \{ [(2\nu_1)^2 + (2\nu_2 - 1)^2] - 2 \frac{(k_I \alpha_1 + k_{II} \alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1(2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 + 2\nu_1(2\nu_2 - 1) \} \times \\ &\quad \times \{ [(2\nu_1)^2 + (2\nu_2 - 1)^2] - 2 \frac{(k_I \alpha_1 + k_{II} \alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1(2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 - 2\nu_1(2\nu_2 - 1) \} \end{aligned} \quad (49)$$

is always positive, negative or zero depending on the frequency ν_1 and ν_2 , saturations η_I , η_{II} and quotient of the conductivities k_{II}/k_I (cf. Fig. 3)

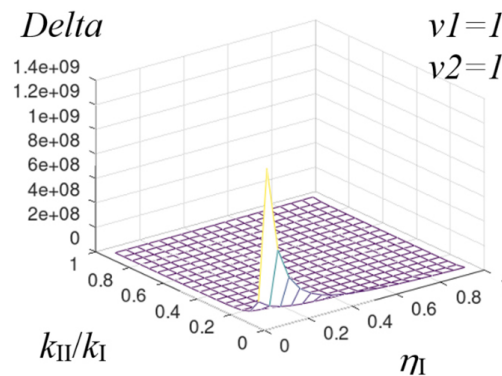


Fig. 3. Graph of the parameter Δ for second analysis

- **Numerical analysis of the roots of the quadratic equation (38) being the characteristic equation of the equation for the amplitude $a_2(z)$ of the even fluctuation transported through a composite building partition and cooperating with two odd remaining Fourier fluctuations**

The values of the determinant Δ and both roots of the quadratic equation (38) has been analysed as functions of the frequencies ν_1 and ν_2 , saturations η_I , η_{II} and quotient of the conductivities k_{II}/k_I (cf. Fig. 4)

$$\begin{aligned} \Delta &= \frac{1}{4}(\gamma_2 + \gamma - \alpha_2^{-1} \alpha^{-1} \beta^2)^2 - \gamma_2 \gamma = \\ &= \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} \{ [(2\nu_1)^2 + (2\nu_2 - 1)^2] - 2 \frac{(k_I \alpha_1 + k_{II} \alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1(2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 \}^2 - \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} [2\nu_1(2\nu_2 - 1)]^2 = \\ &= \frac{1}{\lambda^4} \frac{\langle k \rangle_H^2}{\langle k \rangle^2} \cdot \{ [(2\nu_1)^2 + (2\nu_2 - 1)^2] - 2 \frac{(k_I \alpha_1 + k_{II} \alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1(2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 + 2\nu_1(2\nu_2 - 1) \} / \times \\ &\quad \times \{ [(2\nu_1)^2 + (2\nu_2 - 1)^2] - 2 \frac{(k_I \alpha_1 + k_{II} \alpha_2)^2}{\langle k \rangle_H^2} \cdot \left[\frac{2\nu_1(2\nu_1 + 2\nu_2 - 1)}{(2\nu_2 - 1)^2 - 4\nu_1^2} \right]^2 - 2\nu_1(2\nu_2 - 1) \} \end{aligned} \quad (50)$$

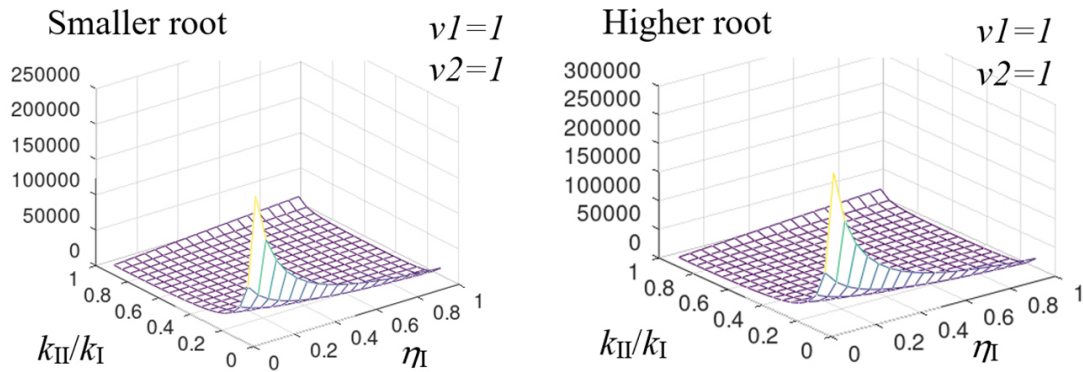


Fig. 4. Graphs of the smaller and higher root for second analysis

For the frequencies $\nu_1 = 1$ and $\nu_2 = 1$ being analysed, the determinant Δ and the both roots of the quadratic equation

$$\Delta > 0, \quad R_1 = B - \sqrt{\Delta} > 0, \quad R_2 = B + \sqrt{\Delta} > 0 \quad (51)$$

are positive. It means that the biquadratic equation (42) has four roots, two positive and two negative:

$$\Delta > 0, \quad R_{1+} = \sqrt{B - \sqrt{\Delta}} > 0, \quad R_{1-} = -\sqrt{B - \sqrt{\Delta}} < 0, \quad R_{2+} = \sqrt{B + \sqrt{\Delta}} > 0, \quad R_{2-} = -\sqrt{B + \sqrt{\Delta}} < 0 \quad (52)$$

Thus, the general integral of the ODE for the amplitude $a_2(z)$ has a form

$$a_2(z) = \omega\left(\frac{z}{\lambda}\right) = C_{1+} e^{r_{1+} \frac{z}{\lambda}} + C_{1-} e^{r_{1-} \frac{z}{\lambda}} + C_{2+} e^{r_{2+} \frac{z}{\lambda}} + C_{2-} e^{r_{2-} \frac{z}{\lambda}} \quad (53)$$

for any real constants C_{1+} , C_{1-} , C_{2+} , C_{2-} , possible to be determined from the boundary values (34) or from an appropriately formulated Cauchy problem for a homogeneous ODE of the 4th order.

REMARKS AND FINAL CONCLUSIONS

After the analysis, it is possible to conclude only about the dependence on the exponential function. Thus, in the case examined, the pure exponential damping occurs what enables to control only boundary conditions. In the graphs presented above, the area with the peak is the case of a difference between the material properties. It means that one of the materials is a very good conductor and the other one – very poor, thus, it occurs a huge damping of fluctuations within the composite walls. The higher the peak, the higher the absolute values of the roots (higher difference between the roots – they become distant from each other).

For both pairs of the analysed frequencies, i.e. for $(\nu_1, \nu_2) = (5, 6)$ as well as for $(\nu_1, \nu_2) = (1, 1)$ and for very high number of frequency pairs for which such analysis had been performed by making graphs of three forms presented above, it has been confirmed the hypothesis:

Research hypothesis: The determinant Δ and the both roots of the quadratic equation (43)

$$\Delta > 0, \quad R_1 = B - \sqrt{\Delta} > 0, \quad R_2 = B + \sqrt{\Delta} > 0 \quad (54)$$

are positive. It means that the biquadratic equation (42) has four roots: two positive and two negative:

$$R_{1-} = -\sqrt{B - \sqrt{\Delta}} < 0, \quad R_{2+} = \sqrt{B + \sqrt{\Delta}} > 0, \quad R_{2-} = -\sqrt{B + \sqrt{\Delta}} < 0 \quad (55)$$

Hence, the general integral of the homogeneous 4th order ODE (36) for the amplitude $a_2(z)$ (the even Fourier fluctuation being transported along with the two remaining Fourier fluctuations through the composite building partition) has a form

$$a_2(z) = \omega\left(\frac{z}{\lambda}\right) = C_{1+} e^{\eta_+ \frac{z}{\lambda}} + C_{1-} e^{\eta_- \frac{z}{\lambda}} + C_{2+} e^{\eta_+ \frac{z}{\lambda}} + C_{2-} e^{\eta_- \frac{z}{\lambda}} \quad (56)$$

Conclusion: The boundary thermal load in the form of the even Fourier fluctuation, being transported along with the two remaining Fourier fluctuations through the composite building partition, will never evoke a thermal pulsation of the composite partition.

RESUMÉE

In the paper, the surface localization of composite heat transfer equations has been used to the analysis of selected properties of the thermal behaviour of the boundary effect in periodic composites. This model is an equivalent reformulation of the parabolic equation of heat conduction. This reformulation consists of:

- the single equation for an averaged temperature;
 - a finite set of equations for amplitudes of tolerance fluctuations describing thermal phenomena occurring on a discontinuity surface – these equations are fulfilled only on discontinuity surfaces;
 - an infinite set of equations for amplitudes of Fourier fluctuations (coefficients of the Fourier development) describing a behaviour of transport of boundary temperature disturbances through an area occupied by a composite – these equations are fulfilled only inside the areas of material (thermal) homogeneity.
1. The homogeneous part of the set of equations for the Fourier amplitudes constitutes a description of a so-called thermal boundary effect, i.e. description of a part of transport of the boundary thermal fluctuations, not burdened by an influence of the averaged temperature, through an area in the composite.
 2. The transport of triple thermal Fourier impulses consisting of one odd and two even Fourier fluctuations, mutually cooperating with each other has been analysed in the paper. The literature distinguishes between two scalar parameters of damping of these fluctuations – a rotational and exponential damping. In the studies of the boundary effect behaviour, so far undertaken (correctors in the asymptotic homogenization or tolerance modelling) and not using the surface localization method, **only the exponential damping was considered**.
 3. It has been proved numerically in the paper that, for selected types of fluctuation, the hypothesis formulated in the end of the paper is true, i.e. a conductor is able to induce only a typical, exponential damping of the transported boundary disturbances of temperature. Their sinusoidal pulsations in the direction transversal to the periodicity directions are not possible.
 4. In a special case, the satisfaction of the aforementioned hypothesis means that the exponential damping of the boundary impulse is maximal for components with very different material properties (for selected triplets of fluctuations). Such situations are corresponded with a characteristic peak in the graphs presented in the work.

The paper shows that during the transmission of three Fourier pulses through the tested composite, rotational damping was excluded. This result is so surprising and on this account it is present in the paper.

Authors' contributions

Conceptualization: D.K.; methodology: D.K.; validation: D.K. and A.A.; formal analysis: D.K. and A.A.; investigation: A.A.; data curation: D.K. and A.A.; writing – original draft preparation: A.A.; writing – review and editing: D.K.; visualization: A.A.; supervision: D.K.; project administration: D.K.; funding acquisition: D.K.

All authors have read and agreed to the published version of the manuscript.

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TRANSPORT WSPÓŁPRACUJĄCYCH ZE SOBĄ TRÓJEK TERMICZNYCH FLUKTUACJI FOURIERA PRZEZ DWUFAZOWĄ PERIODYCZNĄ PRZEGRODĘ WARSTWOWĄ

STRESZCZENIE

W pracy analizowano zjawisko efektu brzegowego w przegrodzie budowlanej zbudowanej z dwuskładnikowego kompozytu warstwowego. Przyjęto dwuwymiarowy model takiej przegrody, w którym efekt brzegowy jest opisywany układem równań różniczkowych zwyczajnych. W pracy numerycznie wykazano, że dla wybranych typów fluktuacji spełniona jest hipoteza sformułowana na końcu pracy, tj. że przegroda reaguje na obecność brzegowej paczki fluktuacji złożonej z jednej parzystej i dwóch nieparzystych fluktuacji jedynie typowym, wykładniczo tłumionym przenoszeniem tych brzegowych fluktuacji. Nie są możliwe ich sinusoidalne pulsacje w kierunku poprzecznym do kierunków periodyczności. Tłumienie wykładnicze jest maksymalne dla składników o bardzo zróżnicowanych własnościach materiałowych składników (wartości parametrów k_{II}/k_I i η_I bliskie zeru). Takim sytuacjom odpowiada charakterystyczny pik wykresów zamieszczonych w pracy.

Słowa kluczowe: przewodnictwo cieplne, efektywne przewodnictwo cieplne, zjawisko efektu brzegowego, przegroda budowlana