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ORIGINAL PAPER

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THE USE OF THE SURFACE LOCATION APPROACH IN THE MODELING OF PERIODICALLY NONHOMOGENEOUS SLENDER VISCO-ELASTIC BEAMS

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ABSTRACT

The aim of the study is to develop an analytical reformulation the known theory for thin periodically nonhomogeneous viscoelastic beams, which should to form the basis for the planned research of the full beam dispersion relation. The methodology applied in the paper is similar to that described in paper *Tolerance Modelling of Vibrations and Stability for Periodic Slender Visco-Elastic Beams on a Foundation will Damping. Revisiting* by Jarosław Jędrysiak, published in scientific journal "Materials" in 2020, but it differs in the way of the use of tolerance modeling in the beam theory. The mathematical tool of considerations is a special choice of the micro-macro decomposition of the beam deflection. It is based on a certain regularization of the displacement field (in the small neighborhood of discontinuity surfaces) and results a certain reformulations of the classical beams theory. Obtained model equations is an alternative proposal for model equations obtained in mentioned paper by Professor Jędrysiak as a certain approximation for periodically nonhomogeneous viscoelastic beams theory as a result of the original tolerance modeling developed. The standardization of the beam deflection field proposed in the presented paper allows us to use the infinite Fourier expansion for deflection field in any region occupied by a homogeneous periodic composite material, which can be modeled in a typical way using the virtual work principle for commonly used beam constrains. The deflection field of the beam is written in the form of an infinite Fourier series, using periodically distributed region of material homogeneity of the beam. Applied method can be viewed as an attempt to use an infinite number of shape functions in the tolerance modeling but at the same time as an equivalent reformulation of the equations of the beam vibration theory. The obtained system of equations is an infinite system of ordinary differential equations for infinitely many Fourier coefficients in the mentioned Fourier expansion of the beam deflection field with respect to the base described in the surface location method.

Key words: periodically nonhomogeneous beams, tolerance modeling, Fourier expansion

INTRODUCTION

The paper proposes a certain two-stage method of modeling periodic composite beams. In the first stage corresponds to the typical procedure, based on the of the virtual displacements principle, leading to the beam theory. The second stage is a special implementation of the constraints method developed in Woźniak and Wierzbicki (2000) as the tolerance modeling technique. The problem investigated in this paper is in fact identical to the problem perfectly described in Jędrysiak (2020). Both works, however, differ in the way of implementing of the tolerance modeling in the second stage. The alternative approach proposed in this paper leads to an infinite system of model equations and this infinite number of equations appears to be a drawback

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of the proposed approach. Each of these shape functions, however, describes a different type of periodic beam deflection fluctuations, and they all complete an orthogonal Fourier basis. This method of realization of the tolerance modeling has already been used in thermal conductivity of periodic composites as the surface localized tolerance modeling (cf. Kula & Wierzbicki, 2019). In order to make the paper self-consistent, both stages of the modeling of periodic slender visco-elastic composites will be described.

A graphic illustration of the considered beam in the special case of two-phased periodic composite material is located in the figure.

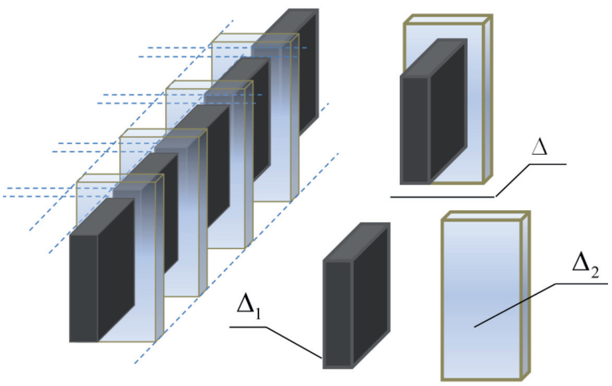

Fig. A graphic illustration of the considered two-phased periodic beam together with distinguished periodicity cell Δ and materially homogeneous cell parts $\Delta_1 \equiv \equiv \Omega_1 \cap \Delta$ and $\Delta_2 \equiv \Omega_2 \cap \Delta$

For given: length of a beam L, $L > 0$ and beam axis range $\mathbb{L} = [0, L] \times \{0,0\} \times \{0,0\}$, for given beam width $a = a(x)$ and beam height $h = h(x)$, $x \in [0, L]$, consider the undeformed beam region as

$$\Omega = \{(x, y, z) : -a(x)/2 \le y \le a(x)/2, \\ -h(x)/2 \le z \le h(x)/2, x \in [0, L]\} \tag{1}$$

Assume that geometrical properties of the beam, represented by height function $h = h(\cdot)$ and weight function $a = a(\cdot)$, as well as foundation properties, represented by Winkler coefficient $k = k(\cdot)$, damping parameter $c = c(\cdot)$ and mass density of foundation $\mu = \mu(\cdot)$, which should be some restrictions of l – periodic functions, $0 < l < L$, i.e. functions defined in R and satisfying condition $f(x) = f(x + l)$ for any x, $x + l \in R$. Positive real number l not necessary need to be the smallest period of $k = k(\cdot)$, $c = c(\cdot)$ and $\mu = \mu(\cdot)$ but it, together with independence on (y, z) of $h = h(\cdot)$ and $a = a(\cdot)$, determines the Δ – periodicity property of the considered beam for $\Delta = \bigcup_{x \in [0,l]} \{x\} \times [-a(x), a(x)] \times [-h(x), h(x)]\}$. Hence, elasticity modulus $E(\cdot, y, z)$, visco-elasticity modulus $B(\cdot, y, z)$ and mass density $\rho(\cdot, y, z)$ should be assumed to be Δ – periodic functions.

In the subsequent considerations we shall also assume that discontinuity surfaces Γ separating beam sub-regions occupied by the same homogeneous material are perpendicular to the beam axis. Hence

$$\Delta_k = \bigcup_{x \in \mathcal{L}_k} \{x\} \times [-a(x), \\ a(x)] \times [-h(x), h(x)]\} \tag{2}$$

for $k = 1, ..., \kappa$ (and given positive integer κ) is a sequence of the repetitive cell sub-regions occupied by the same material. It uniquely determines the partition $\{\mathcal{L}_k\}_{k=1,...,\kappa}$ of a repetitive cell $\mathcal{L} \equiv [0, l]$ onto a finite sequence of materially homogeneous sub-intervals $\mathcal{L}_k$, $k = 1, ..., \kappa$. They will be referred to as the maximal materially homogeneous subintervals of $\mathcal{L}$. The assumed form of the surface network distribution Γ we treat as the simplest form that allows the use of beam constraints as a modeling approach results beam equations satisfied with an acceptable engineering accuracy.

The displacement vector field will be represented by $u(x, y, z) = [u_1, v](x, y, z)$, $v \equiv [u_2, u_3]$, $(x, y, z) \in \Omega$. At the same time stress and strain components we are to represent by vectors $s \equiv [\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{zx}]$, $\in \equiv [\in_{xx}, \in_{yy}, \in_{zz}, \in_{xy}, \in_{yz}, \in_{zx}]$, respectively, interrelated by the attached constitutive relations. $s = C \in$ written under the material matrix

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \tag{3}$$

Finally q stands for the total loading in the z-axis direction.

OUTLINE OF THE MODELING PROCEDURE OF SLENDER BEAMS

Following Jędrysiak (2020), the virtual work principle will be assumed in the form

$$\begin{aligned}&\int_{[0,L]} dx \int_{-a/2}^{a/2} dy \int_{-h/2}^{h/2} dz[\rho(\ddot{u}_1\bar{u}_1^{\,T} + \ddot{v}\,\bar{v}^{\,T}) + s\,\bar{\in}] = \\ &= \int_{[0,L]} dx \int_{-a/2}^{a/2} dy \int_{-h/2}^{h/2} dz[q - (kv + \mu\ddot{v} + c\dot{v})]\bar{v}^{\,T}\end{aligned} \tag{4}$$

which should be hold for any $(0, u_2, w) \in V \subset \subset [C^1(\Omega)]^3$ and:

- the kinematic beam-type of the displacement constraints

$$\begin{aligned}&B \equiv \{u = (u_1, u_2, u_3) \in [C^1(\Omega)]^3 : \\ &u_1 \equiv 0,\ u_2(x, y, z) = -z\partial w(x), \\ &u_3 \equiv w(x) \in C^4([0, L])\ \}\end{aligned} \tag{5}$$

where $\partial \equiv \partial / \partial x$.

- the virtual displacement space introduced in the form

$$\begin{aligned}&V = \{u = u(\cdot, t) \in [C^1(\Omega)]^3 : u\,|_{\partial\Omega} = \\ &= u_{\partial\Omega} \text{ and } jump(\sigma_n)\,|_{\Gamma} = 0\ \} \cap B :\end{aligned} \tag{6}$$

for unilateral stress components

$$\begin{aligned}\sigma_n \equiv [&\sigma_{xx}n_x + \sigma_{xy}n_y + \sigma_{zx}n_z, \sigma_{yx}n_x + \sigma_{yy}n_y + \\ &+ \sigma_{yz}n_z, \sigma_{zx}n_x + \sigma_{yz}n_y + \sigma_{zz}n_z]^T\,|_{\Gamma\cup\partial\Omega}\end{aligned} \tag{7}$$

externally normal to boundaries of materially homogeneous components of Ω (defined in the set r $\Gamma_{reg} \subset \Gamma \cup \partial\Omega$ of regular points in $\Gamma \cup \partial\Omega$) and jumps of these components σ_n on $\Gamma_{reg} \cap \Gamma$ denoted by $jump(s_n)\,|_{\Gamma}$.

In the notation (4) the symbols of iterated integrals are not commutative and are treated as operators acting on functions represented by symbols placed directly on the right side of each integral.

We are to treat the virtual work principle (4) as a starting point of the modeling procedure. Note that the coefficients of the stiffness matrix depend here only on the Young modulus (E) and the Poisson ratio (v), i.e.:

$$\begin{aligned}&C_{11} = C_{22} = C_{33} = \frac{E}{1+v} \\ &C_{12} = C_{23} = C_{13} = \frac{vE}{(1+v)(1-v)} \\ &C_{44} = C_{55} = C_{66} = \frac{vE}{2(1+v)}\end{aligned} \tag{8}$$

In the case of homogeneous materials the virtual work principle (4) has been used as a fundamental formulation of the laws of mechanics being a starting point for procedures which via various restrictions imposed onto a displacement virtual space (6) leads to formulations of various beam theories. The micro-macro tolerance approximation has been used as a tool to enable the application of proportionality the displacement longitudinal component and the deflection gradient in the modeling of inhomogeneous beams (Woźniak & Wierzbicki, 2000).

Similarly, as in Jędrysiak (2020), we shall introduce:

- the strain–displacement relation

$$e = \partial w \tag{9}$$

where $e \equiv \in_{33}$ is interpreted as beam strain field;

- the stress–strain relation

$$s = s_0 + Ee + B\dot{e} \tag{10}$$

In Eq. (10) $s_0 \equiv s_{xx}$ and $E = E(x)$ and $B = B(x)$ are interpreted as Λ – periodic functions being elasticity and visco-elasticity modulus, respectively. Consequently, one can arrive at the governing equations of slender beam in the form

$$\partial\partial m - \partial(f\partial w) + \mu\ddot{w} - j\partial\partial\ddot{w} + kw + \hat{\mu}\ddot{w} + c\dot{w} = q \tag{11}$$

together with constitutive equation

$$m = d\partial\partial w + b\partial\partial\dot{w} \tag{12}$$

Substituting Eq. (11) into Eq. (12) governing equations of their periodic beams can be written as

$$\begin{aligned}&\partial\partial(d\partial\partial w) + \partial\partial(b\partial\partial\dot{w}) - \partial(g\partial w) + \\ &+ \mu\ddot{w} - j\partial\partial\ddot{w} + kw + \mu\ddot{w} + c\dot{w} = q\end{aligned} \tag{13}$$

In Eqs. (11), (12) and (13) stiffness of the beam: bending $d = d(x)$ and visco-elastic $b = b(x)$ as well as mass density $\mu = \mu(x)$ and the rotational mass inertia $j = j(x)$, defined as

$$\begin{aligned} d(x) &= \int_{-a/2}^{a/2} dy \int_{-h/2}^{h/2} dz[E(x,y,z)z^2] \\ b(x) &= \int_{-a/2}^{a/2} dy \int_{-h/2}^{h/2} dz[B(x,y,z)z^2] \\ \mu(x) &= \int_{-a/2}^{a/2} dy \int_{-h/2}^{h/2} dz[\rho(x,y,z)] \\ j(x) &= \int_{-a/2}^{a/2} dy \int_{-h/2}^{h/2} dz[\rho(x,y,z)z^2] \end{aligned} \tag{14}$$

being periodic functions as well as axis force g given by

$$g = \int_{-a/2}^{a/2} dy \int_{-h/2}^{h/2} dz[s_0(x,y,z)] \tag{15}$$

have also been used.

Note that the role of virtual displacement space V for virtual work balance (4) takes over for (13) an another space W of virtual deflections $\overline{w} = \overline{w}(x)$. However, focusing on the realization of the aim of the paper, the second stage of modeling will be carried out in the next section, similarly as in Jędrysiak (2020), using the orthogonalization method originally taken into account by a creator of the tolerance modeling – Czesław Woźniak (Woźniak & Wierzbicki, 2000).

SURFACE LOCALIZED DECOMPOSITION OF THE DEFLECTION FIELD

In the framework of surface localization approach to the tolerance modeling (Kula & Wierzbicki, 2019), it is assumed that a basic physical field w (beam displacement field or here: beam deflection field) is decomposed onto a sum of continuously differentiable fields referred to as long-wave part (L-part) and short-wave part (S-part), respectively:

$$w \equiv w_L + w_S \tag{16}$$

in which the long-wave term w_L and the short-wave term w_S, are restricted by condition $\langle w \rangle = \langle w_L \rangle$ (or equivalently $\langle w_S \rangle = 0$). Under the definition formulated in the previous sequence terms w_L and w_S are not given uniquely. Indeed, if $w_L = v_L + v_S$ and $\langle v_S \rangle = 0$ then $w_L = v_L + v_S$ for $v_S = w_S + v_S$ is also (L, S)-decomposition.

In order to define (L, S)-decomposition and Fourier representation of displacement field usually used in the surface localization approach we are to introduce successively:

– The composite piece-wise averaging $\prec f \succ = \prec f \succ (x)$ of $f:[0,L] \to R$ such that if $\mathcal{L}_k$ is a unique maximal materially homogeneous subinterval of $\mathcal{L}$ and $\mathcal{L}_{sk} \equiv sl + \mathcal{L}_k$, $k = 1, 2, ..., \kappa$, $s = 1, 2, ...$ Then

$$\prec f \succ_k (x) \equiv \frac{1}{|\mathcal{L}_{sk}|} \int_{\mathcal{L}_{sk}} f(\xi) d\xi \quad \text{provided that} \quad x \in \mathcal{L}_{sk} \tag{17}$$

together with the piecewise constant network of macropoints $\prec x \succ_f \in \mathcal{L}_{sk}$ such that $f(\prec x \succ_f) = \prec f \succ (x)$ for any $x \in \mathcal{L}_{sk}$. The unique macropoint placed in $\mathcal{L}_{sk} \equiv sl + \mathcal{L}_k$ will be denoted by l_{sk}, $k = 1, ..., \kappa$, $s = 1, 2, ...$ It is mean that $l_{sk} = \prec x \succ_f$ if $x \in \mathcal{L}_{sk}$ and $l_{sk} \in \mathcal{L}_{sk}$. Note that if $\tilde{x}, \overline{x} \in [0,L]$ are placed in the same maximal materially homogeneous subinterval $\mathcal{L}_{sk}$ of $\mathcal{L}$, $k = 1, ..., \kappa$, $s = 1, 2, ...$, then $\prec f \succ_k (\tilde{x}) = \prec f \succ_k (\overline{x})$.

– $\mathcal{L}$ – mean value $\langle f \rangle (x)$ of $f: R \to R$ in defined by

$$\langle f \rangle (x) \equiv \int_{\mathcal{L}(x) \equiv x + \mathcal{L}} f(\xi) d\xi = \sum_{\substack{k=1,...,\kappa \\ s=1,2,...,}} \eta_{ks} \prec f \succ_k (\prec l_{sk} \succ_f)$$

$$\eta_{ks} \equiv \frac{|\mathcal{L}_{sk} \cap \mathcal{L}|}{|\mathcal{L}|} \tag{18}$$

for $x \in \mathcal{L}_{\sim 1} \cup ... \cup \mathcal{L}_{\kappa} = \mathcal{L} \equiv [0,l]$ any $x \in [0,L]$ for which $\mathcal{L}(x) \subset [0,L]$.

– The trace $\Gamma_{[0,L]}$ of the collection of surfaces $\Gamma \cap \Omega$ separating regions occupied by composite components defined by $\Gamma_{[0,L]} \times \{0,0\} \times \{0,0\} = \Gamma \cap \mathbb{L} \subset \Omega$ (in the case, investigated in the present paper, in which $\Gamma \subset \Omega$ is perpendicular to the beam axis

$\mathbb{L}$, $\Gamma_{[0,L]}$ is orthogonal projection trace of $\Gamma \subset \Omega$ onto $\mathbb{L}$).

– ε – ribbon $\gamma_\varepsilon(\Gamma_{[0,L]})$ of defined by

$$\gamma_\varepsilon(\Gamma_{[0,L]}) \equiv \{x \in [0,L] \setminus \Gamma_{[0,L]} : dist\{x; \Gamma_{[0,L]}\} < \varepsilon\} \tag{19}$$

where on ε – (L, S)-decomposition $w = w_{L,\varepsilon} + w_{S,\varepsilon}$ of the deflection field w determined by a pair ($w_{L,\varepsilon}$, $w_{S,\varepsilon}$) $w_{S,\varepsilon} = w - w_{L,\varepsilon}$, and by quadratic ε – regularization $w_{L,\varepsilon}$ of w

$$w_{L,\varepsilon}(x) = \begin{cases} w(x_\varepsilon) + 0.25(\xi_\varepsilon - d_\varepsilon)^2 w_\Gamma & \text{for} \quad x \in \gamma_\varepsilon(\Gamma_{[0,L]}) \\ w(x) & \text{for} \quad x \in [0,L] \setminus \gamma_\varepsilon(\Gamma_{[0,L]}) \end{cases} \tag{20}$$

where $\xi_\Gamma \equiv dist\{x, \Gamma_{[0,L]}\}$ and $\xi_\varepsilon \equiv \varepsilon - \xi_\Gamma > 0$.

– Limit (L, S)-decomposition $w = w_L + w_S$ of u determined by a pair (w_L, w_S) introduced by limit passage $(w_L, w_S) \equiv \lim_{\varepsilon \searrow 0}(w_{L,\varepsilon}, w_{S,\varepsilon})$.
– Collection of orthogonal (in $L^2(\mathcal{L}_k)$) and l – periodic Fourier basis $g^r{}_k = g^r{}_k(\cdot) \in C^2(\mathcal{L}_k)$, $r = 0, 1, 2, \ldots$, which for any $k = 1, \ldots, \kappa$ consists of the infinite sequence (indexed by $r = 0, 1, 2, \ldots$) of Fourier fluctuations defined in R and which for $r > 0$ is formed by $\mathcal{L}$-oscillating functions, i.e. l – periodic functions such that $\langle g^r \rangle = 0$ and $g^r \equiv 1$ for $r = 0$ with respect to which beam deflection $w = w(x)$ is possible to be represented in any $\mathcal{L}_{sk} \subset [0,L]$, $k = 1, \ldots, \kappa$, by Fourier expansions

$$\begin{aligned} w = w(x) = a_{0k} + g^r{}_k(x) a_{rk}, \\ x \in \mathcal{L}_{sk}, \quad k = 1, \ldots, \kappa \end{aligned} \tag{21}$$

(summation convention with respect to $r = 1, 2, \ldots$ holds) with coefficients (Fourier amplitudes)

$$a_{0k} = \prec w \succ_k, \quad a_{rk} = \prec w g^r \succ_k \tag{22}$$

Remark. In expansion (21) Fourier coefficients (22) are constant in every $\mathcal{L}_{sk}$ but strongly depend on integers s and k. At the same time in expansion (21) Fourier fluctuations $g^r{}_k = g^r{}_k(\cdot)$ do not depend on integer parameter s and obviously depend on the enumerator r of Fourier terms.

THE USE OF SURFACE LOCALIZED VERSUS OF TOLERANCE MODELING FOR SLENDER PERIODIC BEAMS

If we deal with two phased ($\kappa = 2$) l – periodic slender beam, related to the equation of motion (13), l – periodic Fourier basis is considered in the form familiar with that used in Kula and Wodzyński (2020). It can be written in the form

$$g^r{}_L(x) = \begin{cases} \frac{\lambda}{2}\left\{1 - \alpha_1\left[1 + \cos\left(2\pi r\left(\frac{x}{\lambda\eta^{\mathrm{I}}} + 1\right)\right)\right]\right\} & \text{for} \quad -\lambda\eta^{\mathrm{I}} \le x \le 0 \\ \frac{\lambda}{2}\left\{1 - \alpha_1\left[1 + \cos\left(2\pi r\left(\frac{\bar{x}}{\lambda\eta^{\mathrm{I}}} + 1\right)\right)\right]\right\} & \text{for} \quad 0 \le x \le \lambda\eta^{\mathrm{II}}, \bar{x} = 0 \end{cases}$$

$$g^r{}_L(x) = \begin{cases} \frac{\lambda}{2}\left\{1 - \alpha_2\left[1 + \cos\left(2\pi r\left(\frac{\bar{x}}{\lambda\eta^{\mathrm{II}}} - 1\right)\right)\right]\right\} & \text{for} \quad -\lambda\eta^{\mathrm{I}} \le x \le 0, \bar{x} = 0 \\ \frac{\lambda}{2}\left\{1 - \alpha_2\left[1 + \cos\left(2\pi r\left(\frac{x}{\lambda\eta^{\mathrm{II}}} - 1\right)\right)\right]\right\} & \text{for} \quad 0 \le x \le \lambda\eta^{\mathrm{II}} \end{cases}$$

$$\bar{g}^r_{ODD}(x) = \begin{cases} -\frac{\lambda}{2}\cos\left((2r-1)\pi\left(\frac{x}{l^{\mathrm{I}}} + 1\right)\right) & \text{for} \quad -l^{\mathrm{I}} \le x \le 0 \\ -\frac{\lambda}{2}\cos\left((2r-1)\pi\left(\frac{x}{l^{\mathrm{II}}} - 1\right)\right) & \text{for} \quad 0 \le x \le l^{\mathrm{II}} \end{cases} \tag{23}$$

where $\lambda_\eta{}^{\mathrm{II}} \equiv diam(\mathcal{L}_2)$, $\eta^{\mathrm{I}} + \eta^{\mathrm{II}} = 1$, and microstructural parameter λ is taken as equal to the periodicity parameter l, $\lambda = l$. In other cases, for example. If $\lambda = 2l$, it is necessary to apply the realization of the procedure for extending the representation of the deflection field to the Fourier expansion valid along the whole beam length another than that presented in this section of the paper. Parameters $\alpha_1 = 1 / (\eta_{\mathrm{I}} + 2\eta_{\mathrm{II}})$, $\alpha_2 = 1 / (2\eta_{\mathrm{I}} + \eta_{\mathrm{II}})$ are uniquely chosen in such a way that oscillating conditions $\langle g^r{}_L \rangle = \langle g^r{}_R \rangle = 0$ are satisfied. Such shape functions have already been used in many works (Wodzyński, Kula & Wierzbicki, 2018; Kula, 2019; Kula & Wierzbicki, 2019; Wodzyński, 2020). In Kula and Wodzyński (2020), $g^r{}_L(x)$, $g^r{}_L(x)$ and $\bar{g}^r_{ODD}(x)$, are referred to as left even, right even and odd Fourier fluctuations, respectively. Even Fourier fluctuations are properly constructed and appropriately adapted Fourier bases used in Eqs. (21) and (22), and odd Fourier fluctuations complement them to complete l – periodic Fourier basis given by Eq. (23) for the deflection field w.

Now we are to reformulate governing equations (11) to the form adopted to new form of by the virtual slender beam deflection

$$\begin{aligned}U \equiv \{&\omega = \omega(x) = a_{0k} + g^r{}_{ODD}(x) o_{rk} + g^r{}_L(x) l_{rk} + \\ &+ g^r{}_R(x)\, r_{rk} : \\ a_{0k} &= \eta_{\mathrm{I}} \prec \omega \succ_{\mathrm{I}} + \eta_{\mathrm{II}} \prec \omega \succ_{\mathrm{II}} \in R, \\ o_{rk} &= \eta_{\mathrm{I}} \prec \omega g^r{}_{ODD}(x) \succ_{\mathrm{I}} + \eta_{\mathrm{II}} \prec \omega g^r{}_{ODD}(x) \succ_{\mathrm{II}} \in R, \\ l_{rk} &= \eta_{\mathrm{I}} \prec \omega g^r{}_L(x) \succ_{\mathrm{I}} + \eta_{\mathrm{II}} \prec \omega g^r{}_L(x) \succ_{\mathrm{II}} \in R, \\ r_{rk} &= \eta_{\mathrm{I}} \prec \omega g^r{}_R(x) \succ_{\mathrm{I}} + \eta_{\mathrm{II}} \prec \omega g^r{}_R(x) \succ_{\mathrm{II}} \in R, \\ &x \in \mathcal{L}_k, k = 1, ..., \kappa\}\end{aligned} \tag{24}$$

Following Jędrysiak (2020), we shall to consider and solve the problem written as

$$\begin{aligned}&\partial\partial m - \partial(n \partial w) + \mu \ddot{w} - j \partial\partial \ddot{w} + k w + \hat{\mu} \ddot{w} + \\ &\quad + c \dot{w} - q = reaction, \quad reaction \perp U\end{aligned} \tag{25}$$

in the framework of which we are obliged to formulate the system of equations for Fourier coefficients (Fourier amplitudes) A_0, O_r, L_r, R_r in the new deflection representations

$$w = w(x) = A_0 + g^r{}_{ODD}(x) O_r + g^r{}_L(x) L_r + g^r{}_R(x)\, R_r \tag{26}$$

which are proposed in this paper. It is also worth mentioning that Fourier amplitudes used in the above representation are constant in sufficiently small homogeneous sub-regions $\mathcal{L}_{sk} \subset [0, L]$ of the region occupied by the periodic composite which is investigated as a periodic beam. Hence they can be treated as locally slowly varying, (Ostrowski, 2020). The investigation of Fourier amplitudes A_0, O_r, L_r, comes down to applying the formal orthogonalization procedure

$$\begin{aligned}\langle \partial\partial m - \partial(n\partial w) + \mu \ddot{w} - j\partial\partial \ddot{w} + kw + \hat{\mu}\ddot{w} + c\dot{w} - q \rangle = 0 \\ \langle (\partial\partial m - \partial(n\partial w) + \mu \ddot{w} - j\partial\partial \ddot{w} + kw + \hat{\mu}\ddot{w} + c\dot{w} - q) g^r{}_{ODD} \rangle = 0 \\ \langle (\partial\partial m - \partial(n\partial w) + \mu \ddot{w} - j\partial\partial \ddot{w} + kw + \hat{\mu}\ddot{w} + c\dot{w} - q) g^r{}_L \rangle = 0 \\ \langle (\partial\partial m - \partial(n\partial w) + \mu \ddot{w} - j\partial\partial \ddot{w} + kw + \hat{\mu}\ddot{w} + c\dot{w} - q) g^r{}_R \rangle = 0\end{aligned} \tag{27}$$

for any integer r and deflection represented by Eq. (26). Similar form of the orthogonalization procedure are taken into account in Jędrysiak (2020). Just mentioned orthogonalization procedure results model equations of the investigated periodic beam

$$\begin{aligned}&\partial\partial[\langle d\rangle \partial\partial U + \langle d\partial\partial(g^A)\rangle V_A + \langle b\rangle \partial\partial \dot{U}]] - \partial[\langle n\rangle \partial U + \langle n\partial g^A\rangle V_A] - \\ &-\langle j\rangle \partial\partial \ddot{U} + \langle k\rangle U + (\langle \mu\rangle + \langle \hat{\mu}\rangle)\ddot{U} + \langle c\rangle \dot{U} - \langle q\rangle = 0 \\ \\ &\langle d\partial\partial g^B\rangle \partial\partial U + \langle d\partial\partial(g^A)\partial\partial(g^B)\rangle V_A + \langle (b\partial\partial g^A \partial\partial g^B\rangle \dot{V}_A - \\ &+\langle n\partial g^B\rangle \partial U + \langle n\partial g^A \partial g^B\rangle V_A + \langle \mu g^A g^B\rangle \ddot{V}_A - \\ &-\langle j\partial\partial(g^A) g^B\rangle \ddot{V}_A + \langle k g^A g^B\rangle V_A + \langle \hat{\mu} g^A g^B\rangle \ddot{V}_A + \\ &+\langle c g^A g^B\rangle \dot{V}_A - \langle q g^B\rangle = 0\end{aligned} \tag{28}$$

where $g^A \in \{g^r{}_{ODD}, g^r{}_L, g^r{}_R\}$ for $A = r$ and $g^B \in \{g^r{}_{ODD}, g^r{}_L, g^r{}_R\}$ for $B = r$. Summation deals indicators r as well as the three types of elements of Fourier basis $g^A \in \{g^r{}_{ODD}, g^r{}_L, g^r{}_R\}$ .It should be emphasize once again that, as in the whole volume of the paper, the indicator r runs over positive integers and after the restriction of the summation range in Eq. (28), to $r = 1, ..., N$ reduces expansion (24) to a partial Fourier sum. This situation can be interpreted formally as a result of the application of original tolerance modeling procedure with N tolerance shape functions.

FINAL REMARKS

Equation (28) are treated as the essential result of the presented paper. A detailed analysis of the equations and the announced pearl dispersion analysis of the considered beams is the subject of further research. Since Eq. (28) are the alternative equivalent for equations obtained in Jędrysiak (2020) and therefore they have been in a form which makes it easier to see components which disappear when used instead of the tolerance shape functions of the corresponding components constructed in this paper Fourier series for beam deflection. It should be emphasized that the application of Eq. (28) to solve and investigate specific problems is much more difficult because instead of a finite system of equations for the fluctuation amplitudes obtained as a result of tolerance modeling, we are dealing here with an infinite set of equations for

Fourier amplitudes. On the other hand, Eq. (28) are not merely an approximation of the exact equations of slender beams but represent their equivalent reformulation.

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WYKORZYSTANIE METODY LOKALIZACJI POWIERZCHNIOWEJ W MODELOWANIU CIENKICH BELEK LEPKOSPRĘŻYSTYCH

STRESZCZENIE

Celem pracy jest opracowanie analitycznego przeformułowania znanej teorii cienkich periodycznie niejednorodnych belek lepkosprężystych, które powinno stanowić podstawę do przeprowadzenia planowanych badań pełnej relacji dyspersji dla tego typu belek. Metodologia zastosowana w pracy jest podobna do zaproponowanej w artykule *Tolerance Modelling of Vibrations and Stability for Periodic Slender Visco-Elastic Beams on a Foundation will Damping. Revisiting* Jarosława Jędrysiaka, opublikowanej w czasopiśmie naukowym „Materials" w 2020 roku, ale różni się sposobem wykorzystania modelowania tolerancyjnego wymienionych belek. Matematycznym narzędziem rozważań jest specjalny wybór mikro-makro rozkładu ugięcia wiązki. Polega ona na pewnej regularyzacji pola przemieszczeń (w niewielkim sąsiedztwie powierzchni nieciągłości) i skutkuje pewnymi przeformułowaniami klasycznej teorii belek. Uzyskane równania modelowe są alternatywną propozycją równań modelowych otrzymanych we wspomnianej pracy profesora Jędrysiaka, w której przedstawił przybliżone sformułowanie równań takich belek, wykorzystując klasyczne modelowanie tolerancyjne. Zaproponowana w prezentowanej pracy regularyzacja pola ugięcia belki pozwala nam na użycie nieskończonego rozwinięcia Fouriera obowiązującego dla ugięcia w dowolnym obszarze zajmowanym przez jednorodny periodyczny materiał kompozytowy, który może być modelowany typowym sposobem z wykorzystaniem zasady prac wirtualnych dla powszechnie stosowanych więzów belkowych. Pole ugięcia belki zapisywane jest w postaci nieskończonego szeregu Fouriera, wykorzystującego periodycznie rozłożone obszary jednorodności materiałowej belki. Zastosowana metoda może być postrzegana jako próba wykorzystania nieskończonej liczby funkcji kształtu w modelowaniu tolerancyjnym, ale jednocześnie jako równoważne przeformułowanie równań teorii drgań belek. Otrzymany układ równań jest nieskończonym układem równań różniczkowych zwyczajnych dla nieskończenie wielu współczynników Fouriera w wymienionym rozwinięciu Fouriera pola ugięcia belki względem bazy opisanej w metodzie powierzchniowej lokalizacji.

Słowa kluczowe: belki periodycznie niejednorodne, modelowanie tolerancyjne, szeregi Fouriera