

NATURAL VIBRATIONS OF A TOWER STRUCTURE IN MASS-CHANGING CONDITIONS DUE TO THE ICING

Wiesław Kowalski, Ewelina Świerczek

University of Agriculture in Krakow

Abstract. The study analyzed the impact of additional tower structure masses on the change of the frequency of its natural vibrations; it also made attempts to mathematical description of this impact. Considerations apply to the tower supporting the lighting system of the city stadium in Ostrowiec Świętokrzyski. The source of additional masses is the icing (frost, rime) and non-structural components constituting the technical equipment of the tower. The analysis was made by modeling by means of the Finite Element Method (FEM) of the structure with the assumption of a one parameter variation of its mass (i.e., the additional mass of all components, structural and non-structural, changes in proportion to one parameter, which is the outer surface of the element on which the ice layer is deposited). Solving the problem of natural vibrations, for subsequent models, representing different intensity of tower's icing, the following natural frequencies have been established. Thus, the increase of the ice-layer thickness on the surfaces from 0 to 2.4 cm caused the reduction of basic natural frequencies by a value, consecutively: for $f_1 \rightarrow 26.9\%$; for $f_2 \rightarrow 27.2\%$. These values are important from the point of view of technical applications. The findings allow to formulate a postulate that, in the analysis of tower's structure susceptibility to gusts of wind, structure's mass variability resulting from its possible icing was taken into consideration.

Key words: quasi-truss, lattice pylon, transmission tower, wind-induced resonance, icing

INTRODUCTION

Tower type structures often rise to great heights. They often rise above the objects in their neighborhood. In this situation, tower structures are particularly exposed to the wind, as there is no shield in their natural environment enclosure; particularly in the open area. The impact of air traffic on the object, in addition to static pressure, can have a dynamic character. We are talking here about gusts of wind or other effects caused by undisturbed airflow around the object. In such situation, the process of structural design requires an

Corresponding author: Wiesław Kowalski, University of Agriculture in Krakow, Department of Rural Building, 24/28 Mickiewicza Ave., 59-130 Krakow, e-mail: w.kowalski@ur.krakow.pl

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analysis of the structure's vulnerability on the gusts of wind, and in special situations, it is necessary to analyze the dynamic response of the building (in mechanics of structures the terms such as "Benard-Kármán vortices" or "flutter" are known [Ge and Tanaka 2000, Ge et al. 2000, Zhang et al. 2003]). The problems of structural engineer do not end there. The dynamic reaction of the object is in fact determined by its dynamic characteristics, whereas the frequency and forms of natural vibrations depend, among other things, on the masses of structure's elements and their distribution in the structure [He 1987, Fekr and McClure 1998, Clough and Penzien 2003, Fengli et al. 2010], and are not always easy to determine. Tower type structures, rising high, periodically change their mass, along with the growing process of icing of bars (freezing dew), the accumulation of rime frost and sometimes also with the frozen wet snow (Fig. 1).



Fig. 1. The cross-construction on Giewont Mountain in winter time

The accumulation of ice on parts of the building structures, vehicles and machines, as a constant phenomenon in the cold climate conditions for many years has been an area of concern for the world of science and technology [Macklin 1962, Makkonen 1981]. The problem here is the question of monitoring the objects for the early prediction of dangerous conditions, associated with excessive accumulation of ice [Zaharov 1984, Podrezov et al. 1987, Lehký et al. 2001, Harstveit et al. 2005, Lehký and Sabata 2005, Vaculik and Rampl 2005], the avoidance of excessive development of phenomena of this type, for example by the use of anti-adhesive coatings [Shigeo et al. 2003, Kimura et al. 2004], and the development of procedures and computational techniques for reliable estimation of the icing scale as an additional burden on the structure [PN-87/B-02013, ISO 12494, Fikke et al. 2007, Xie and Sun in 2012, Yang et al. 2012]. The problem of the icing of the objects is of practical importance in regions with cold climates, also in the mountains, and for instance in sea environment, where it is combined with high humidity [Makkonen 1984, 1987, Gates et al. 1986, Hørjen 1989].

In dynamic issues, which are the subject of this discussion, in general case the icing creates two types of problems:

- it is a source of an extra mass, that changes the dynamic characteristics of the structure, thereby interfering in its "vulnerability" on resonance (for example, as already mentioned, due to the gusts of wind) [Eliasson and Thorsteins 1996, Clough and Penzien 2003],
- the cracking and the peeling masses of ice, forces the vibration of the structure. This phenomenon is of particular importance for lattice power pylons, where the peeling ice is a kind of impulse, causing free vibrations of overhead electric conductors, additionally magnified by the gusts of wind, which leads to pylon vibrations induced by rocking of overhead lines [Fekr and McClure 1998, Battista et al. 2003, Havard and Dyke 2005, Kálmán et al. 2007, Marzaneh (ed.) 2008, Fengli et al. 2010, Chen et al. 2014].

In the work there were carried out simulation calculations, aimed at determining the extent of the reduction in the frequency of the tower's structure vibrations, with the increase of its mass as a result of the installation of additional technical equipment, and mainly due to the growing icing layer on the structure. The findings obtained in the course of work are very significant from the cognitive point of view. Above all, they are a contribution to the discussion about the structure's vulnerability on the gusts of wind and the risk of error when assessing it in variable mass conditions. The problem is of particular importance in the design and technical diagnostics of GSM, radio and television relay towers, often localized at high altitudes, even in the mountains. Structures located in such severe climate, meet particularly favorable conditions for the ice to deposit; in such conditions they are also particularly exposed to the wind.

MATERIALS AND METHODS

Model specificity

The tower, analyzed in this work, is not located in the mountains; namely it is the supporting tower for the lighting system in sports stadium in Ostrowiec Świętokrzyski. However, it has many of the typical characteristics for the structure of broadcast-towers type of quasi-truss:

- it is a slender structure, with a considerable height (approx. 48 meters),
- is largely "full of" bars, also with non-structural ones, with a large lateral surface on which ice can be deposited (approx. 933 m²),
- it has a low basic weight (approx. 15,290 kg of structural bars plus 5,983 kg of complementary elements, eg. platforms made of WEMA-latticework), which means that the impact of the icing on its percentage increase can be significant,
- it is the type of structure with low damping (for steel frames and trusses a fraction of critical damping equals $\zeta = 5\%$ or less), so that wind-induced resonance problems has here special significance.

The tower, under consideration, is made of 881 bars of different cross-sections and different joint-types; having in mind the clarity of and to protect the reader from too much information, the detailed description of the characteristics of successive bars and joints has been omitted.

Main structural elements have been joined by screws through special flanges and gusset plates. Additional elements, such as a ladder with fingerplate or platform's guard rails have been welded. Both types of connections create fixed coupling which does not allow for mutual displacement.

The technical description implies [Szeszuła 1990], that 50 lamps are located on a specially designed support frame. Each lamp with weapons weighs 15 kg. Therefore, the total weight of the lighting is 750 kg.

The shape of the tower and its basic dimensions are represented in Figure 2. The FEM model of the structure, using technical documentation of the object [Szeszuła 1990], has been constructed, utilizing computer program Autodesk Robot Structural Analysis. The following assumptions were adopted:

1. The following variants of the model are of frame character, which gives freedom in the modeling of joints. In the real structure the bars have been screwed to each other, with the help of gusset plates; in many joints such a situation does not give freedom of rotation (Fig. 3).

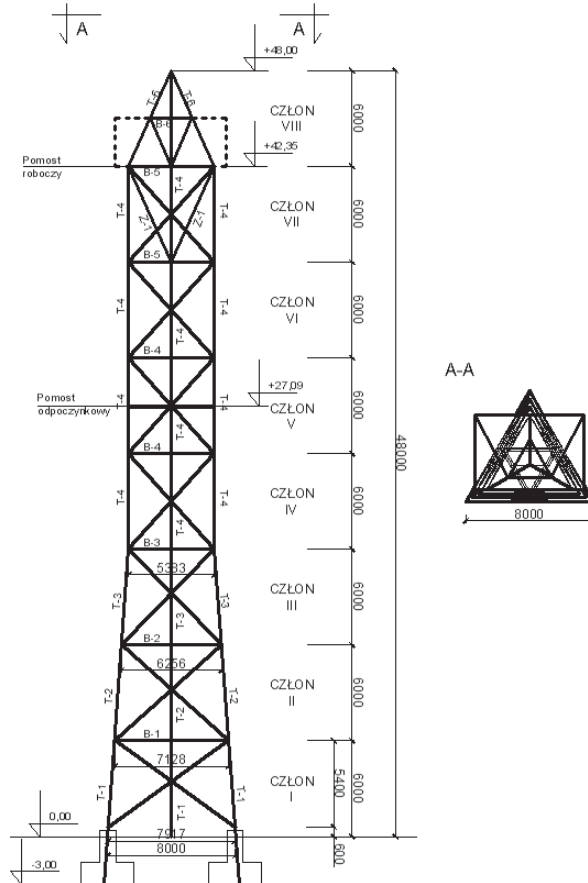


Fig. 2. Shape and basic dimensions of analyzed tower



Fig. 3. Photographs of typical tower's joints

2. Additional, non-structural inertia of the object, associated with the installed lighting, is included in the model as concentrated masses (that do not have rotational inertia) in the joints of the supporting frame.

3. Additional, non-structural inertia of the object, associated with the icing of the structure, is included in the model in the form of a mass distributed evenly along bars and evenly on the surface of platforms (therefore it has a rotational inertia).

4. The increase in the mass due to the icing, in the following variants of the model, is of a one-parameter character, i.e., the additional mass of all elements, structural and non-structural, change in proportion to one parameter, which is the outer surface of the element on which the ice layer is deposited.

5. The value of the specific mass density of the icing has been assumed on the basis of PN-87/B-02013.

6. Three other variants of the model (vide infra) correspond to the characteristic values of ice thickness for three zones of climatic division present in Poland [PN-87/B-02013].

7. The vulnerability of the ground in the foundation of the tower has been neglected.

8. The material of the structure operates within the linear-elastic range.

Several variants of the model, corresponding to the mass gain, as a result of the installation of lighting and the icing of varied intensity, have been prepared. And so:

- model 1 no additional mass is taken into account (only the basic mass of the tower, that is approx. 21,273 kg),
- model 2 includes an additional mass resulting from the mounted lightings (all at once), but with no icing (total model mass equal to 22,023 kg),
- model 3 includes an additional mass resulting from the mounted lightings and from ice-layer of thickness 12 mm (total model mass equal to 29,029 kg),
- model 4 includes an additional mass resulting from the mounted lightings and from ice-layer of thickness 18 mm (total model mass – approx. 31,794 kg),
- model 5 includes an additional mass resulting from the mounted lightings and from ice-layer of thickness 24 mm (total model mass equal to 34,067 kg).

Typical shape of the FEM model of the structure is presented in Figures 4–7.

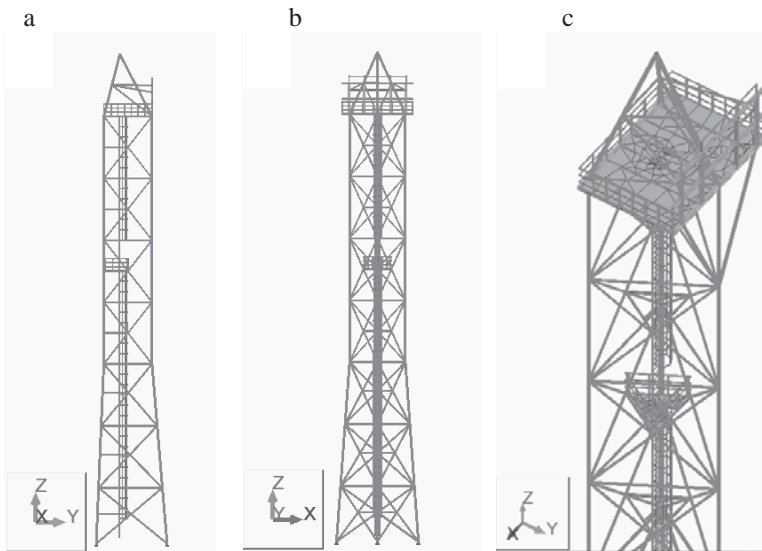


Fig. 4. Model of the tower and its relation to the axis of adopted frame of reference: a – from the side, b – from the front, c – in an axonometric projection

Natural vibrations

The easiest way to present theoretical basis for the description of natural vibration's issue [Clough and Penzien 2003] is on the basis of the so-called "displacement method", starting from the equation of system's free vibration (with many dynamic degrees of freedom):

$$[M] \cdot \ddot{\underline{y}} + [C] \cdot \dot{\underline{y}} + [K] \cdot \underline{y} = \underline{0} \quad (1)$$

where: $[M]$, $[C]$, $[K]$ – square matrices characterizing the dynamic properties of the vibrating system (in order: a matrix of inertia, damping and stiffness),

$\ddot{\underline{y}}$, $\dot{\underline{y}}$, \underline{y} – vectors: of acceleration, velocity and displacement (relative once) of the vibrating system.

Dynamic eigenvalue problems, in the dynamics of structures, come down to determine the circumstances, in which the equation of free vibration system, without the element representing the damping, can have nonzero solution. Thus, in equation (1) the damping is omitted, and the equation looks like this:

$$[M] \cdot \ddot{\underline{y}} + [K] \cdot \underline{y} = \underline{0} \quad (2)$$

Assuming the harmonic form of vibrations:

$$\underline{y} = \underline{A} \cdot \sin(\omega \cdot t + \varphi) \quad (3)$$

$$\ddot{\underline{y}} = \frac{d^2 \underline{y}}{dt^2} = -\omega^2 \cdot \underline{y} \quad (4)$$

where: A – amplitude of vibration,
 ω – angular frequency of vibrations,
 φ – phase shift angel,
 t – time as a variable in the vibration process,

we get a relationship:

$$-\omega^2 \cdot [M] \cdot \underline{y} + [K] \cdot \underline{y} = \underline{0} \quad (5)$$

Multiplying both sides by a matrix of system's vulnerability $[F]$ (from the left side) we get:

$$-\omega^2 \cdot [F][M] \cdot \underline{y} + [F][K] \cdot \underline{y} = \underline{0} \quad (6)$$

$$\text{However, since } [F][K] = [I] \quad (7)$$

$$\text{we get } \left\{ [F] \cdot [M] - \frac{1}{\omega^2} \cdot [I] \right\} \cdot \underline{y} = \underline{0} \quad (8)$$

or in another form, when assuming $\lambda = \frac{-1}{\omega^2}$

$$\{ [F] \cdot [M] - \lambda \cdot [I] \} \cdot \underline{y} = \underline{0} \quad (9)$$

This is a dynamic eigenvalue problem. The task comes down to solving the system of homogeneous linear equations. In all variants of the model of analyzed tower it is a system composed of 6,426 equations which corresponds to the number of 1,071 joints with assigned mass, each with six independent degrees of freedom.

RESULTS

For adopted five models of the analyzed tower, consecutively, ten lowest frequencies of natural vibrations have been evaluated. The results of the calculations are presented in Table 1.

Figure 5 summarizes a degree of consecutive natural frequencies Θ_i reduction as a result of the change in the mass of the tower's vibrating system caused by the installation of the lamps. Figure 6 shows a degree of consecutive natural frequencies Θ_{24} reduction as a result of the change in the mass of the tower's vibrating system caused by icing thickness of 24 mm. It should be noted that in the first case (Fig. 5) the results obtained for model 2 are related to the results for model 1; in the second case (Fig. 6) the results of model number 5 were related to the results for model number 2.

Table 1. Designated ten lowest natural vibration's frequencies of the tower's models

Model number	1	2	3	4	5
Source of mass	Structure	Structure + technical equipment	Structure + technical equipment + icing	Structure + technical equipment + icing	Structure + technical equipment + icing
Ice-layer thickness [mm]	0	0	12	18	24
Form of vibrations	Natural frequency, f (Hz)				
1	1.1272	1.0831	0.8882	0.8248	0.7910
2	1.1372	1.0961	0.8954	0.8327	0.7984
3	3.3978	3.2123	2.7639	2.5528	2.4596
4	3.6447	3.6446	3.6433	3.6428	3.6425
5	4.6187	4.6184	4.6133	4.4273	4.1919
6	4.8009	4.7265	4.7076	4.5239	4.3532
7	6.2251	6.2250	4.7467	4.6206	4.6183
8	6.6475	6.6161	5.0969	4.8713	4.7887
9	7.0152	6.9859	6.2251	6.2250	6.2249
10	7.5431	7.3663	7.1296	6.7032	6.3710

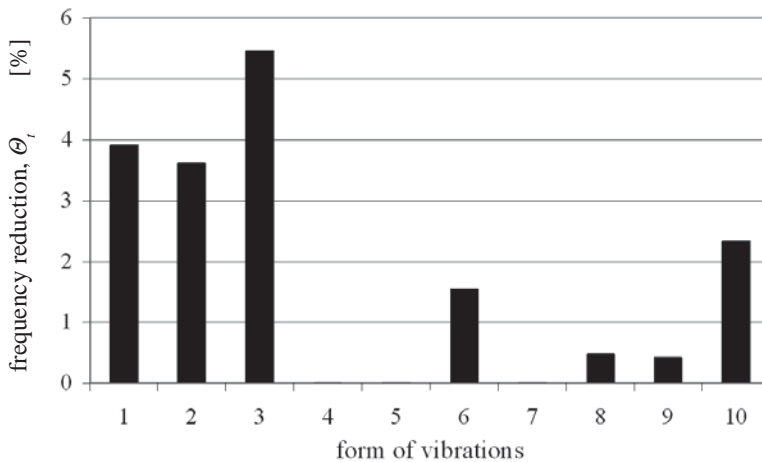


Fig. 5. A degree of natural frequency reduction of the tower model Θ_r with an increase in mass caused by the installation of the lamps; for the first form of vibration $\Theta_r = 3.91\%$

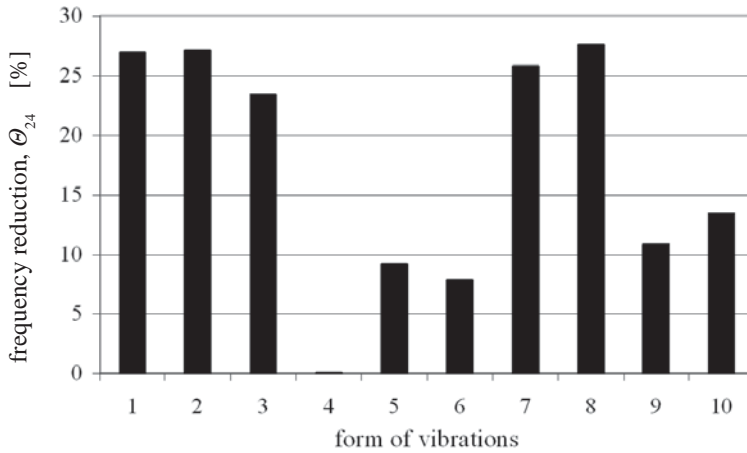


Fig. 6. A degree of natural frequency of the tower model θ_{24} reduction with the icing thickness of 24 mm with respect to the structure free of ice; for the first form of vibration $\theta_{24} = 26.97\%$

An attempt has been made at mathematical description of the relationship between the natural frequency and the ice-layer thickness of structure’s elements, for the following forms of vibrations. Due to the nature of this relationship, an approximation of the obtained results was made by the function:

$$f_N(d) = \frac{A}{\left[1 + \left(\frac{d}{B}\right)^C\right]^D} + E \tag{10}$$

where: d – ice-layer thickness [mm],
 A, B, C, D, E – model coefficients (for calibration).

The above formula has this important property, that the increase in the ice-layer thickness is always accompanied by a decrease in the value of frequency, which corresponds to a physical interpretation of the phenomenon. In the case of some forms of natural vibrations, in this analyzed case, an approximation can be made by means of simple polynomials, of second or third order, with very good results (see Fig. 7, 8, 13). However, in some cases, these polynomials are increasing functions (in some intervals), which corresponds to the situation when an increase in mass causes an increase in natural frequency. From a mathematical point of view, such description can be perceived as correct, but for physical reasons this is an absurd. For presentation purposes, an example of such situation is shown in Figure 11, for the fifth form of vibrations. The same applies to frequencies sixth and ninth.

The results of the approximations show respectively Figures 7–16.

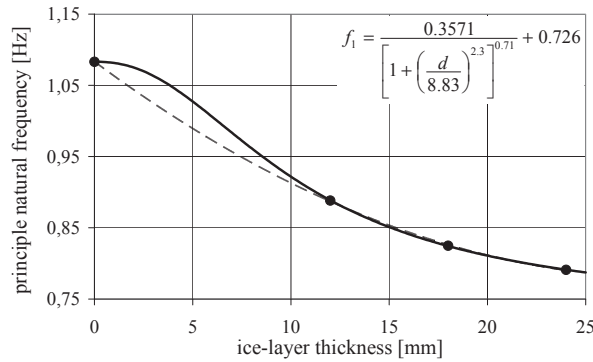


Fig. 7. Principle natural frequency of the model and its dependence on the ice-layer thickness; $R^2 = 1$. The dotted line represents a polynomial relationship $f_1 \leftarrow d$ in the form of $f_1(d) = 0.0003 \cdot d^2 - 0.0205 \cdot d + 1.0833$; $R^2 = 0.999$

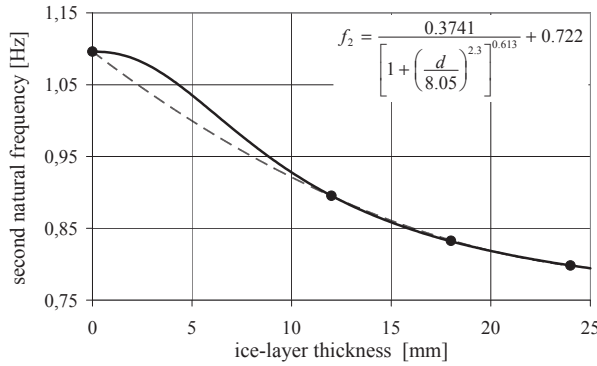


Fig. 8. Second natural frequency of the model and its dependence on the ice-layer thickness; $R^2 = 1$. The dotted line represents a polynomial relationship $f_2 \leftarrow d$ in the form of $f_2(d) = 0.0004 \cdot d^2 - 0.0211 \cdot d + 1.0962$; $R^2 = 1$

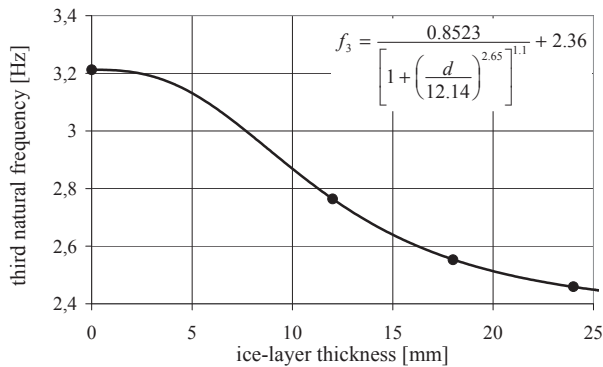


Fig. 9. Third natural frequency of the model and its dependence on the ice-layer thickness; $R^2 = 0.999$

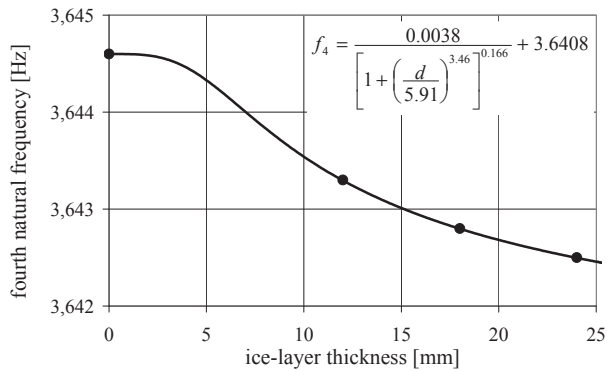


Fig. 10. Fourth natural frequency of the model and its dependence on the ice-layer thickness; $R^2 = 1$

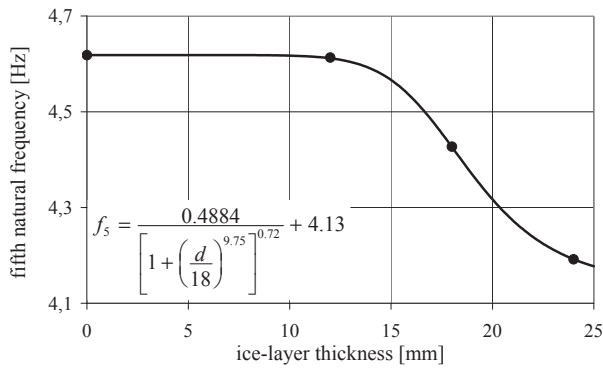


Fig. 11. Fifth natural frequency of the model and its dependence on the ice-layer thickness; $R^2 = 0.996$. The dotted line represents a polynomial relationship $f_5 \leftarrow d$ (unacceptable due to the physical interpretation of the phenomenon) in the form of $f_5(d) = 0.0004 \cdot d^3 - 0.003 \times d^2 + 0.0291 \cdot d + 4.6184$; $R^2 = 1$

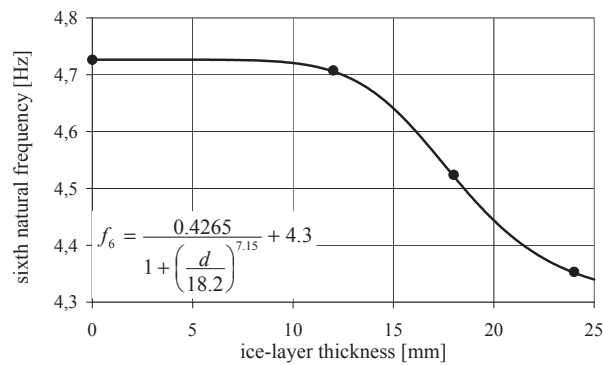


Fig. 12. Sixth natural frequency of the model and its dependence on the ice-layer thickness; $R^2 = 1$

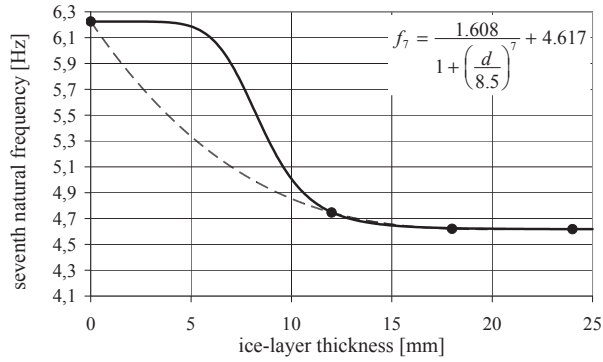


Fig. 13. Seventh natural frequency of the model and its dependence on the ice-layer thickness; $R^2=0.997$. The dotted line represents a polynomial relationship $f_7(d) = -0.0002 \cdot d^3 + 0.0106 \times d^2 - 0.2269 \cdot d + 6.225$; $R^2=1$

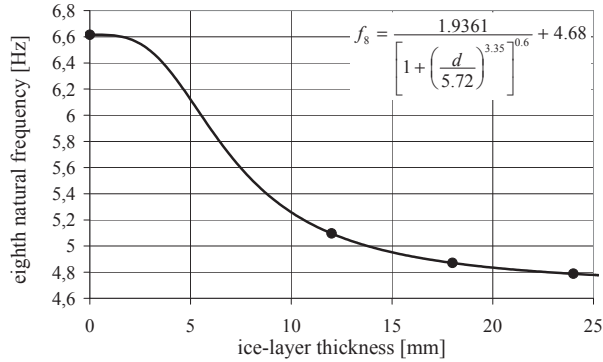


Fig. 14. Eighth natural frequency of the model and its dependence on the ice-layer thickness; $R^2=1$

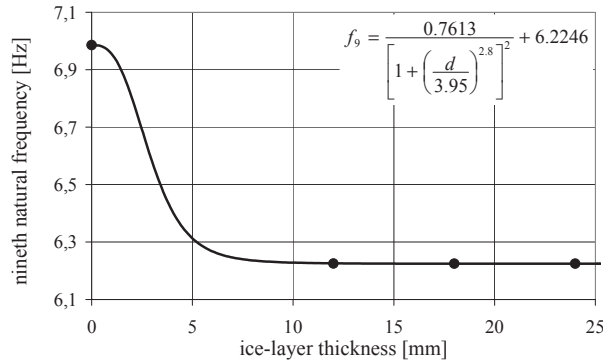


Fig. 15. Ninth natural frequency of the model and its dependence on the ice-layer thickness; $R^2=0.999$

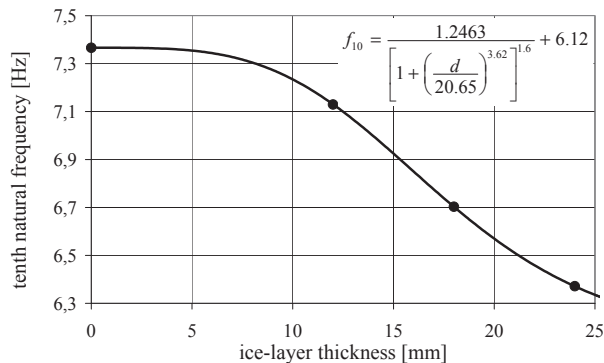


Fig. 16. Tenth natural frequency of the model and its dependence on the ice-layer thickness; $R^2 = 1$

CONCLUSIONS

1. Next natural frequencies arrange themselves in groups with similar values, regardless of the computational model. This is due to the symmetrical construction of the tower.

2. Along with an increase in bars' ice-layer thickness there occurs a decrease in the value of the next natural frequencies of the structure. Among the analyzed 10 basic natural frequencies, only some underwent significant reductions in values: the lowest (first, second and third) and the seventh and eighth. This phenomenon is worth being emphasized.

3. The mass of lighting elements and the icing of bars resulted in a reduction of the fourth natural frequency by only 0.0604%. From a technical point of view, a thesis can be assumed that extra masses do not affect the tower's fourth natural frequency. This is also a surprising result.

4. Additional masses, as above, have little effect (slightly more than 9%) on the frequencies fifth and sixth.

5. An increase in bars' ice-layer thickness, from 0 to 2.4 cm, caused a reduction in natural frequencies, consecutively by: for $f_1 \rightarrow 26.9\%$; for $f_2 \rightarrow 27.2\%$; for $f_3 \rightarrow 23.4\%$; for $f_4 \rightarrow 0.057\%$; for $f_5 \rightarrow 9.2\%$; for $f_6 \rightarrow 7.9\%$; for $f_7 \rightarrow 25.8\%$; for $f_8 \rightarrow 27.6\%$; for $f_9 \rightarrow 10.9\%$ and for $f_{10} \rightarrow 13.5\%$. These values are important from the point of view of technical applications. The bars' icing and the resulting from it decline in natural frequencies should be taken into account when assessing the tower's vulnerability on the gusts of wind.

6. Such functions, as presented below (denoted in the article as No. 10), appear to be very good mathematical models that describe the relationship of consecutive natural frequencies with bar's ice-layer thickness:

$$f_N(d) = \frac{A}{\left[1 + \left(\frac{d}{B}\right)^C\right]^D} + E$$

where: d – ice-layer thickness; A, B, C, D, E – coefficients of the approximation.

The determination coefficient of $0.996 \div 1.0$ indicates a very good match of equations with research results.

7. In some cases, polynomials are good models of wanted dependences, as above.

8. The influence of the ice-layer thickness on the basic natural frequencies [Hz] of the analyzed tower can be described by the dependence:

$$f_1 = \frac{0.3571}{\left[1 + \left(\frac{d}{8.83}\right)^{2.3}\right]^{0.71}} + 0.726$$

where: d – bar's ice-layer thickness [mm].

It should be strongly emphasized, that presented results and conclusions refer to structure under consideration only.

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DRGANIA WŁASNE KONSTRUKCJI WIEŻOWEJ W WARUNKACH ZMIENIAJĄCEJ SIĘ MASY UKŁADU WSKUTEK OBLODZENIA

Streszczenie. W pracy dokonano analizy wpływu dodatkowych mas konstrukcji wieżowej na zmianę częstotliwości jej drgań własnych; dokonano też próby matematycznego opisu tego wpływu. Rozważania dotyczą wieży wsporczej oświetlenia stadionu miejskiego w Ostrowcu Świętokrzyskim. Źródłem dodatkowych mas jest oblodzenie (szron, szadź) oraz elementy niekonstrukcyjne stanowiące wyposażenie techniczne wieży. Analizy dokonano w drodze modelowania metodą elementów skończonych (MES) konstrukcji, przyjmując założenie o jednoparametrycznej zmienności jej masy (to znaczy, że dodatkowa masa wszystkich elementów, konstrukcyjnych i niekonstrukcyjnych zmienia się proporcjonalnie do jednego parametru, którym jest powierzchnia zewnętrzna elementu, na której odkłada się warstwa lodu). Rozwiązując zagadnienie własne drgań dla kolejnych modeli, reprezentujących różne intensywności oblodzenia wieży, wyznaczono wartości kolejnych częstotliwości drgań własnych. I tak, przyrost grubości warstwy lodu na powierzchniach, od 0 do 2,4 cm spowodował redukcję podstawowych częstotliwości drgań własnych o wartości kolejno: dla $f_1 \rightarrow 26,9\%$; dla $f_2 \rightarrow 27,2\%$. Są to istotne wartości z punktu widzenia zastosowań technicznych. Uzyskane wyniki pozwalają na sformułowanie postulatu, aby w analizach podatności konstrukcji wieżowych na porywy wiatru uwzględniana była zmienność masy konstrukcji wynikająca z możliwego jej oblodzenia.

Słowa kluczowe: quasi-kratownica, wieża kratownicowa, słup trakcyjny, rezonans wiatrowy, oblodzenie

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