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A FORCED DAMPED VIBRATIONS OF THE ANNULAR PLATE MADE OF FUNCTIONALLY GRADED MATERIAL

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Abstract. The work deals with the problem of dynamics of bodies made of smart materials in which a characteristic, deterministic and microstructural construction has an influence on the macroscopic properties of the entire system. In particular, the subject of the analysis is the annular plate made of a material with functionally graded properties (FGM). On the microstructural level the structure is constructed from two materials. In the circular direction the structure is periodic. In the radial direction the averaged properties of the structure are functionally variable. The study examined the issue of forced damped vibration. The starting point of discussion is the equation of motion of a thin plate based on Kirchhoff theory. Recently, the outcome of the modeling has been obtained as a partial differential equation for the unknown slowly varying function w^0. This equation, as opposed to output equations of motion with discontinuous and highly oscillating coefficients, has smooth slowly varying coefficients. Hence, they can be solved easily by numerical methods because of its slowly varying coefficient. Moreover, the study shows the exemplary solutions of model equations for the asymptotic model. There is determined the form of the first symmetrical mode shapes for a sample set task parameters and selected the initial-boundary conditions.

Key words: tolerance averaging technique, vibration plates, FGM

INTRODUCTION

The dynamic behavior of the plate with heterogeneous microstructure is described by equations with discontinuous coefficients. In the last decade the new approach to the mathematical modeling of FGM has been proposed. This approach is referred to as the tolerance modeling of FGM and the overview of the results in this field was summarized in Woźniak et al. [2008]. There are two main reasons for applying the tolerance modeling of differential equations as an alternative to the asymptotic non-uniform homogenization. Firstly, for many microheterogeneous media the space distribution of material properties is not uniquely described by means of locally periodic functions. Secondly, the asymp-

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totically homogenized (taking into account the homogenisation of the first order) equations are independent of the microstructure size length parameter. Hence, they are unable to describe the effect of the length scale on the overall behavior of FGM. The tolerance averaging of differential equations overcomes the aforementioned restrictions.

The subject of this paper considers the dumping forced vibration analysis of a thin plate made of functionally graded material. The considered material has periodic properties in the circular direction ξ^1 and slow functionally graded properties in the radial direction ξ^2.

The main objective of the research is to derive and apply a deterministic macroscopic model describing the dynamic behaviour of micro-heterogeneous annular plate made of two components (Fig. 1). The general assumption of the research is that the generalized period λ is sufficiently small comparing to the size of the plate L. The main attention is given to describe the effect of the material distribution on the overall response of the composite.

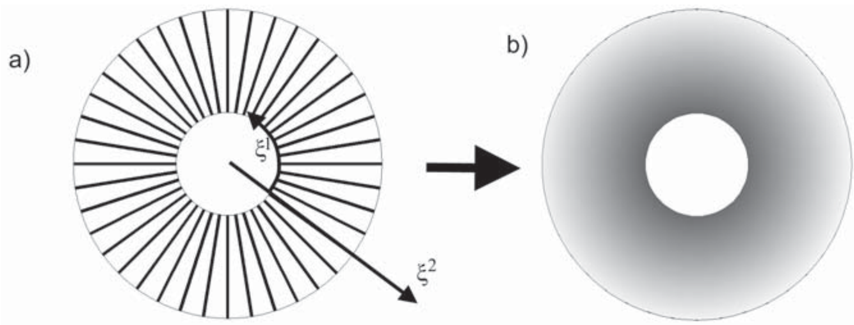

Fig. 1. Scheme (top view) of the analyzed composite: a – microstructure level (black color – ribs, white color – matrix), b – averaged properties of the structure

Rys. 1. Schemat (widok z góry) analizowanego kompozytu: a – poziom mikrostrukturalny (czarny kolor – żebra, biały kolor – matrycę), b – uśrednione własności struktury

The problems of the plates of this kind have been investigated by means of different methods. However, the exact analysis of those plates within solid mechanics is too complicated to constitute the basis for solving most of the engineering problems. Thus, many different approximate modeling methods for functionally graded material plates have been formulated [Rhee 2007, Hui-Shen 2009], The proposed modeling approach is the generalization of the tolerance averaging technique (TAT). This technique was presented in detail in Woźniak and Wierzbicki [2000]. By TAT we can determine the answer of the analyzed structure on the forced of the vibration and the influence of microstructure size on the damping factor.

MODELLING

The general way to create equations of motion is the same as in Michalak and Wirowski [2012] which is discussed in detail. The starting point for modelling is well known equations of linear elasticity theory, which are used to write equations on the micro scale.

The base of modeling procedure are:

– strain-displacements relations

$$\kappa_{\alpha\beta} = -w|_{\alpha\beta} \tag{1}$$

where: $\kappa_{\alpha\beta}$ is curvature, w is displacement field,

– constitutive equations

$$m^{\alpha\beta} = DH^{\alpha\beta\gamma\delta}\kappa_{\gamma\delta} \tag{2}$$

where:

$$H^{\alpha\beta\gamma\delta} = \frac{1}{2}\left\{\begin{array}{l} g^{\alpha\mu}g^{\beta\gamma} + g^{\alpha\gamma}g^{\beta\mu} - \\ +\upsilon\left(\in^{\alpha\gamma}\in^{\beta\mu} + \in^{\alpha\mu}\in^{\beta\gamma}\right) \end{array}\right\} \tag{3}$$

$$D = \frac{E\delta^3}{12\left(1-\upsilon^2\right)} \tag{4}$$

E – Young modulus, δ – thickness of the plate, υ – Poisson number, $\in^{ij}$ – component of Ricci tensor, $g^{\alpha\beta}$ – component of contravariant metric tensor.

After applying formulas (1)–(4) we can write the equation of motion for the band plate under consideration as:

$$m^{\alpha\beta}_{|\alpha\beta} + p - \rho\ddot{w} + B\dot{w} = 0 \tag{5}$$

where: p – the external load, B – dumping coefficient, the symbol "$|$" is the covariant derivative.

Subsequently, we use tolerance averaging technique (TAT) for modeling the dynamic behavior of thin plates, as it is presented in Woźniak and Wierzbicki [2000]. More extensive discussion about TAT and the bibliography containing various examples of applications of this theory can be found in the monograph Woźniak et al. (ed.) [2008]. For the purpose of this paper we shall quote only briefly some of the concepts defined by this theory:

The most important operators and lemates are:

– an averaging operator

$$\langle f\rangle\left(\xi^{\alpha}\right) = \frac{1}{\lambda}\int_{\xi^1-\frac{\lambda}{2}}^{\xi^1+\frac{\lambda}{2}} f\left(y,\xi^2\right)dy \tag{6}$$

where: y is a local coordinate,

– a slowly varying function $F(\cdot) \in SV_{\Delta}(T)$

$$\forall x, y \in \Pi \quad x - y \in \Delta \Rightarrow |DF(x) - DF(y)| < \varepsilon_{DF} \tag{7}$$

where: e_{DF} is a tolerance parameter,

$$DF \in \{F, \nabla F, F, ...\} \tag{8}$$

– the displacement field disjoint (this is the modeling assumption which is called as micro-macro decomposition)

$$w(\cdot,t) = w^0(\cdot,t) + h^A(\cdot)V_A(\cdot,t) \tag{9}$$

where: $w^0(\cdot,t) \in SV_{\Delta}(T),\ V_A(\cdot,t) \in SV_{\Delta}(T)$ are the basic unknowns, and $h^A(\cdot)$ are the known shape functions,

– the most important theorems

$$\langle fF \rangle(x) \cong \langle f \rangle F(x) \tag{10}$$

$$\langle f\nabla(hF) \rangle(x) \cong \langle fF\nabla h \rangle(x) \tag{11}$$

$$\langle f\nabla\nabla(hF) \rangle(x) \cong \langle fF\nabla\nabla h \rangle(x) \tag{12}$$

These definitions and theorems will be used to build the equations of the averaged model. A wider discussion on the derivation and the proof of these associations has been summarized in the monographs Woźniak et al. (ed.) [2008].

The modelling procedure is based on two steps. At first, we put into the equations of motion (5) the assumption of the decomposition of the displacement field (9) and we obtain the equation with N+1 unknowns w^0 i V_A, where A=1 ... N

$$m^{\alpha\beta}_{|\alpha\beta} + p - \rho\left(\ddot{w}^0 + h^A \ddot{V}_A\right) + B\left(\dot{w}^0 + h^A \dot{V}_A\right) = R \tag{13}$$

We obtain the missing equations by orthogonalization method, multiplying the equation of motion by the functions h^A and we get the equations:

$$\langle R \rangle = 0, \left\langle h^A R \right\rangle = 0 \tag{14}$$

After substituting the constitutive equations, the strain-displacements relations, the displacement field disjoint and many mathematical transformations, we get the averaging equations:

$$\left\langle DH^{\alpha\beta\gamma\delta} w^{0}_{|\gamma\delta}\right\rangle_{\}\alpha\beta} + \left(\left\langle DH^{\alpha\beta 11} h^{A}_{|11}\right\rangle V_{A}\right)_{|\alpha\beta} + \left(\left\langle DH^{\alpha\beta 22} h^{A}\right\rangle V_{A|22}\right)_{|\alpha\beta} + \\ +\left\langle \rho \ddot{w}^{0}\right\rangle + \langle B\rangle \dot{w}^{0} + \left\langle Bh^{A}\right\rangle \dot{V}_{A} = \langle p\rangle \tag{15}$$

$$\left\langle h^{A}{}_{|11} DH^{11\gamma\delta}\right\rangle w^{0}_{|\gamma\delta} + \left\langle h^{A}_{|11} DH^{1111} h^{B}_{|11}\right\rangle V_{B} + \left\langle h^{A}_{|11} DH^{1122} h^{B}\right\rangle V_{B|22} + \\ + \left(\left\langle h^{A}{}_{|11} DH^{22\gamma\delta}\right\rangle w^{0}_{|\gamma\delta}\right)_{|22} + \left(\left\langle h^{A} DH^{2211} h^{B}_{|11}\right\rangle V_{B}\right)_{|22} + \left(\left\langle h^{A} DH^{2222} h^{B}\right\rangle V_{B|22}\right)_{|22} + \\ + \left\langle h^{A} \rho h^{B}\right\rangle \ddot{V}_{B} + \left\langle h^{B} B\right\rangle \dot{w}^{0} + \left\langle h^{B} B h^{A}\right\rangle \dot{V}_{A} = \left\langle h^{A} p\right\rangle \tag{16}$$

The coefficients in the above system are described by continuous and slowly varying functions.

THE ASSYMPTOTIC MODEL

A model in which the coefficients in equations describing the annular plate will not depend on the parameter of the microstructure size we call an asymptotic model. If in formulas (15) and (16) we make the passing limit with the parameter of the microstructure,we assume that the cell is infinitely small comparing to the size of the body. Then concerned modules which depend on the parameter λ will tend to zero. Note that if h^A shape functions are dependent on the parameter λ^2, some of the averaged modules in equations (15) and (16) also depends on the microstructure parameter, namely:

$$\left\langle h^{A} DH^{2211}\right\rangle, \left\langle h^{A} DH^{2222} h^{B}\right\rangle, \left\langle h^{A} DH^{2222}\right\rangle, \left\langle h^{A} DH^{2211} h^{B}{}_{|11}\right\rangle,$$

$$\left\langle h^{A} \rho h^{B}\right\rangle, \left\langle DH^{1212} h^{A}{}_{|1} h^{B}{}_{|1}\right\rangle, \left\langle DH^{1212} h^{A}{}_{|1}\right\rangle \cdot \left\langle h^{B} B\right\rangle \left\langle h^{B} B h^{A}\right\rangle$$

Let us note that for the asymptotic model the equations simplified considerably because of the slackness of the system of equations (15) and (16). We can determine the unknown V_A from equation (16) as:

$$V_{A} = -w^{0}{}_{,11} \frac{\left(\left\langle q^{A}{}_{|11} DH^{1111}\right\rangle + \left\langle q^{A}{}_{|11} DH^{2211}\right\rangle_{,22}\right)}{\left(\left\langle q^{A}{}_{|11} DH^{1111} q^{B}{}_{|11}\right\rangle\right)} - w^{0}{}_{,22} \frac{\left(\left\langle q^{A}{}_{|11} DH^{1122}\right\rangle\right)}{\left(\left\langle q^{A}{}_{|11} DH^{1111} q^{B}{}_{|11}\right\rangle\right)} + \\ -w^{0}{}_{,2} \xi^{2} \frac{\left(\left\langle q^{A}{}_{|11} DH^{1111}\right\rangle\right)}{\left(\left\langle q^{A}{}_{|11} DH^{1111} q^{B}{}_{|11}\right\rangle\right)} \tag{17}$$

After many mathematic transformations a 4-th row partial differential equation is built as:

$$\frac{\partial^4 w}{\partial\left(\xi^2\right)^4}A\left(\xi^2\right)+\frac{\partial^3 w}{\partial\left(\xi^2\right)^3}B\left(\xi^2\right)+\frac{\partial^2 w}{\partial\left(\xi^2\right)^2}C\left(\xi^2\right)+\frac{\partial w}{\partial\xi^2}D\left(\xi^2\right)+$$
$$+\frac{\partial^2 w}{\partial t^2}E\left(\xi^2\right)+\frac{\partial w}{\partial t}F\left(\xi^2\right)=G(\xi^2,t) \tag{18}$$

where:

$$A\left(\xi^2\right)=\left\langle DH^{2222}\right\rangle-\frac{\left(\left\langle h^A{}_{|11}DH^{1122}\right\rangle\right)}{\left(\left\langle h^A{}_{|11}DH^{1111}h^B{}_{|11}\right\rangle\right)}\left(\left\langle DH^{2211}h^A{}_{|11}\right\rangle\right)$$

$$B\left(\xi^2\right)=\left(\frac{2}{\xi}\left\langle DH^{2222}\right\rangle+2\left\langle DH^{2222}\right\rangle_{,2}\right)+$$
$$-\left(\frac{\left(\left\langle h^A{}_{|11}DH^{1122}\right\rangle\right)}{\left(\left\langle h^A{}_{|11}DH^{1111}h^B{}_{|11}\right\rangle\right)}\right)\begin{pmatrix}2\left\langle DH^{2211}h^A{}_{|11}\right\rangle_{,2}+\frac{2}{\xi}\left\langle DH^{2211}h^A{}_{|11}\right\rangle+\\ -\xi\left\langle DH^{1111}h^A{}_{|11}\right\rangle\end{pmatrix}-$$
$$+\left(2\left(\frac{\left(\left\langle h^A{}_{|11}DH^{1122}\right\rangle\right)}{\left(\left\langle h^A{}_{|11}DH^{1111}h^B{}_{|11}\right\rangle\right)}\right)_{,2}-\xi\frac{\left(\left\langle h^A{}_{|11}DH^{1111}\right\rangle\right)}{\left(\left\langle h^A{}_{|11}DH^{1111}h^B{}_{|11}\right\rangle\right)_{,2}}\right)\left(\left\langle DH^{2211}h^A{}_{|11}\right\rangle\right)$$

$$C\left(\xi^2\right)=2\left\langle DH^{2211}\right\rangle-\xi^2\left\langle DH^{1111}\right\rangle+\xi\left\langle DH^{2211}\right\rangle_{,2}+\left\langle DH^{2222}\right\rangle_{,22}+\frac{1}{\xi}\left\langle DH^{2222}\right\rangle_{,2}-$$
$$+\left(\frac{\left(\left\langle h^A{}_{|11}DH^{1122}\right\rangle\right)}{\left(\left\langle h^A{}_{|11}DH^{1111}h^B{}_{|11}\right\rangle\right)}\right)\begin{pmatrix}\left\langle DH^{2211}h^A{}_{|11}\right\rangle_{,22}+\frac{2}{\xi}\left\langle DH^{2211}h^A{}_{|11}\right\rangle_{,2}+\\ -\xi\left\langle DH^{1111}h^A{}_{|11}\right\rangle_{,2}-2\left\langle DH^{1111}h^A{}_{|11}\right\rangle\end{pmatrix}-$$
$$+\left(\left(\frac{\left(\left\langle h^A{}_{|11}DH^{1122}\right\rangle\right)}{\left(\left\langle h^A{}_{|11}DH^{1111}h^B{}_{|11}\right\rangle\right)}\right)_{,2}+\xi\frac{\left(\left\langle h^A{}_{|11}DH^{1111}\right\rangle\right)}{\left(\left\langle h^A{}_{|11}DH^{1111}h^B{}_{|11}\right\rangle\right)}\right)\begin{pmatrix}2\left\langle DH^{2211}h^A{}_{|11}\right\rangle_{,2}+\\ +\frac{2}{\xi}\left\langle DH^{2211}h^A{}_{|11}\right\rangle+\\ -\xi\left\langle DH^{1111}h^A{}_{|11}\right\rangle\end{pmatrix}-$$
$$+\begin{pmatrix}\frac{2\left(\left\langle h^A{}_{|11}DH^{1111}\right\rangle\right)}{\left(\left\langle h^A{}_{|11}DH^{1111}h^B{}_{|11}\right\rangle\right)}+\\ -2\xi\left(\frac{\left(\left\langle h^A{}_{|11}DH^{1111}\right\rangle\right)}{\left(\left\langle h^A{}_{|11}DH^{1111}h^B{}_{|11}\right\rangle\right)}\right)_{,2}-\left(\frac{\left(\left\langle h^A{}_{|11}DH^{1122}\right\rangle\right)}{\left(\left\langle h^A{}_{|11}DH^{1111}h^B{}_{|11}\right\rangle\right)}\right)_{,22}\end{pmatrix}\left(\left\langle DH^{2211}h^A{}_{|11}\right\rangle\right)$$

$$
\begin{aligned}
&D\left(\xi^2\right) = -3\xi\left\langle DH^{1111}\right\rangle + \frac{2}{\xi}\left\langle DH^{2211}\right\rangle + \xi\left\langle DH^{2211}\right\rangle_{,22} + \\
&+4\left\langle DH^{2211}\right\rangle_{,2} - \xi^2\left\langle DH^{1111}\right\rangle_{,2} - \frac{1}{\xi}\left\langle DH^{2222}\right\rangle_{,2} \\
&-\xi\frac{\left(\left\langle h^{A}{}_{|11}BH^{1111}\right\rangle\right)}{\left(\left\langle h^{A}{}_{|11}BH^{1111}h^{B}{}_{|11}\right\rangle\right)}\left(\begin{array}{l}\left\langle DH^{2211}h^{A}{}_{|11}\right\rangle_{,22} + \frac{2}{\xi}\left\langle DH^{2211}h^{A}{}_{|11}\right\rangle_{,2} + \\ -\xi\left\langle DH^{1111}h^{A}{}_{|11}\right\rangle_{,2} - 2\left\langle DH^{1111}h^{A}{}_{|11}\right\rangle\end{array}\right) \\
&-\left(\frac{\left(\left\langle h^{A}{}_{|11}DH^{1111}\right\rangle\right)}{\left(\left\langle h^{A}{}_{|11}DH^{1111}h^{B}{}_{|11}\right\rangle\right)} + \xi\left(\frac{\left(\left\langle h^{A}{}_{|11}DH^{1111}\right\rangle\right)}{\left(\left\langle h^{A}{}_{|11}DH^{1111}h^{B}{}_{|11}\right\rangle\right)}\right)_{,2}\right)\left(\begin{array}{l}2\left\langle DH^{2211}h^{A}{}_{|11}\right\rangle_{,2} + \frac{2}{\xi}\left\langle DH^{2211}h^{A}{}_{|11}\right\rangle - \\ +\xi\left\langle DH^{1111}h^{A}{}_{|11}\right\rangle\end{array}\right) - \\
&+2\left(\frac{\left(\left\langle h^{A}{}_{|11}DH^{1111}\right\rangle\right)}{\left(\left\langle h^{A}{}_{|11}DH^{1111}h^{B}{}_{|11}\right\rangle\right)}\right)_{,2} - \xi\left(\frac{\left(\left\langle h^{A}{}_{|11}DH^{1111}\right\rangle\right)}{\left(\left\langle h^{A}{}_{|11}DH^{1111}h^{B}{}_{|11}\right\rangle\right)}\right)_{,22}\left(\left\langle DH^{2211}h^{A}{}_{|11}\right\rangle\right)
\end{aligned}
$$

$$E\left(\xi^2\right) = \langle\rho\rangle,\ F\left(\xi^2\right) = \langle B\rangle,\ G\left(\xi^2,t\right) = \langle p\rangle$$

The coefficients in this equation are continuous, and they can be found by symbolic calculations.

THE NUMERICAL RESULTS

Let us consider the following example: the free vibrations of a thin plate band. This plate is shown in Figure 1a. in circular coordinates. We make some assumptions:
the shape function

$$h(\cdot) = \lambda^2\left(\cos\left(\frac{2\pi\xi^1}{\lambda}\right) + \mathrm{C}\right) \tag{19}$$

where the constant C is obtained from the equation:

$$\langle h\rho\rangle = 0 \tag{20}$$

as

$$\mathrm{C} = \frac{\lambda\xi^2\left(-\rho_1 + \rho_1\cos\left(\frac{2\pi d}{\xi^2\lambda}\right) - \rho_2 - \rho_2\cos\left(\frac{2\pi d}{\xi^2\lambda}\right)\right)}{2\pi\left(\rho_1 d + \rho_2\lambda\xi^2 - \rho_2 d\right)} \tag{21}$$

To numerical calculations there is set the plate of following geometry:

$R_1 = 1$ m, $R_2 = 3$ m, $\lambda = \Pi/100$, $h = 12$ cm, $d = 0.02$ m

In the first case there is examined relation between mode shapes and boundary conditions. It is assumed a decay factor and parabolic forced vibration of the form:

$$\langle p \rangle = 10^6 (x_2 - 1)(x_2 - 3)\sin(2\pi t)e^{-t} \tag{22}$$

It is supposed that the plate consists of two materials of following properties (Table 1).

Table 1. Material properties of the plate
Tabela 1. Właściwości materiałowe płyty

Specyfication Wyszczególnienie	E [GPa]	ρ [kg·m^{-3}]	ν [–]
Matrix	20/200	780/7800	0.3
Ribs	200	7800	0.3

Obtained mode shapes in the case of mutual free support (Fig. 2a) and cantilever (Fig. 2b) are presented below.

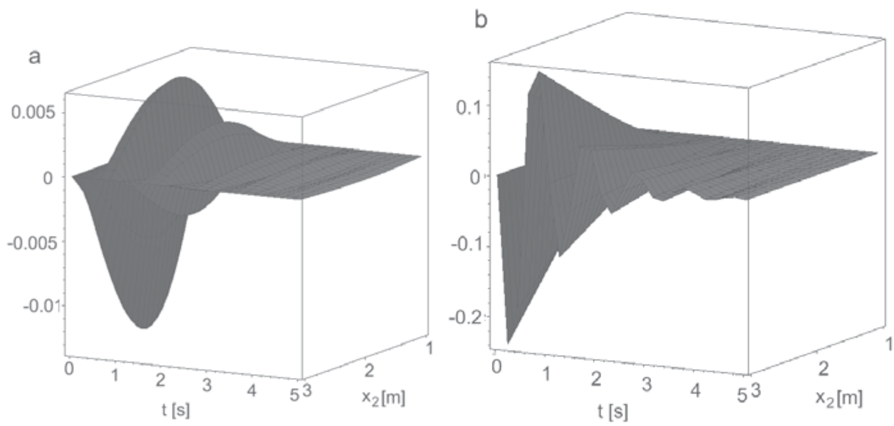

Fig. 2. 3D diagrams that shows deflection of annular plate for different boundary conditions: a – mutual free support, b – cantilever fixed on the inner edge. On the horizontal axis there is presented the time t [s] and the radial coordinate of annular plate x_2 [m], whereby $x_2 = 1$ stands for inner edge and $x_2 = 3$ for outer edge

Rys. 2. Trójwymiarowy diagram przedstawiający ugięcie płyty dla różnych warunków brzegowych: a – obustronne swobodne podparcie, b – wspornik utwierdzony na wewnętrznym brzegu. Na osiach poziomych przedstawiono czas [s] i współrzędną promieniową płyty pierścieniowej x_2 [m], przy czym $x_2 = 1$ oznacza brzeg wewnętrzny, $x_2 = 3$ oznacza brzeg zewnętrzny

The boundary conditions:

– cantilever $\left(w^0\right)_{r=R_1} = 0; \left(\frac{\partial w^0}{\partial r}\right)_{r=R_1} = 0$

– free support $(M_r)_{r=R_2} = 0; (V)_{r=R_2} = \left(Q_r - \frac{1}{r}\cdot\frac{\partial M_{r\varphi}}{\partial \varphi}\right)_{r=R_2} = 0$

$$M_r = -D\left[\frac{\partial^2 w^0}{\partial r^2} + \nu\left(\frac{1}{r}\cdot\frac{\partial w^0}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 w^0}{\partial \varphi^2}\right)\right];$$

where: $Q_r = -D\frac{\partial}{\partial r}\cdot\left(\frac{\partial^2 w^0}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial w^0}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 w^0}{\partial \varphi^2}\right);$

$$M_{r\varphi} = (1-\nu)\cdot D\left(\frac{1}{r}\cdot\frac{\partial^2 w^0}{\partial r\partial\varphi} - \frac{1}{r^2}\cdot\frac{\partial w^0}{\partial\varphi}\right).$$

Subsequently, there is examined the influence of material differences on the mode of dynamic response of the composite plate with functionally graded material properties. It is assumed large (10x) differences between materials of the rib and the matrix. Obtained mode shapes at the time of t = 0.2 ... 4 s are presented below (Fig. 3).

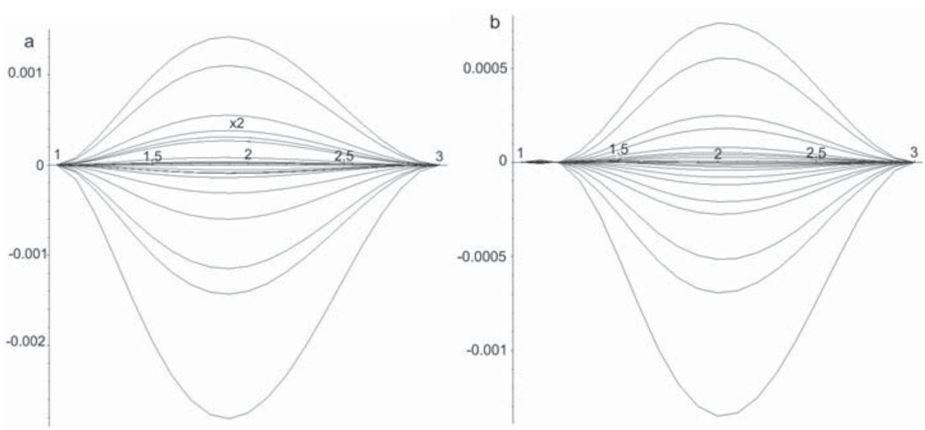

Fig. 3. Deflection of the plate for different material proportions of matrix and ribs: a – E_2/E_1 = 10, ρ_2/ρ_1 = 1, b – E_2/E_1 = 1, ρ_2/ρ_1 = 10. On the horizontal axis there is presented the radial coordinate of the annular plate x_2 [m], whereby x_2 = 1 stands for inner edge and x_2 = 3 for outer edge

Rys. 3. Ugięcie płyty dla różnych proporcji materiałowych: a – E_2/E_1 = 10, ρ_2/ρ_1 = 1, b – E_2/E_1 = 1, ρ_2/ρ_1 = 10. Na osiach poziomych przedstawiono współrzędną promieniową płyty pierścieniowej x_2 [m], przy czym x_2 = 1 oznacza brzeg wewnętrzny, x_2 = 3 oznacza brzeg zewnętrzny

It can be noticed that in the case of plate stiffness change the shape of dynamic response of the system remains the same as for a homogeneous plate. In the case of ribs density modification, the shape of the vibration changes substantially showing functionally variable character of the plate. In comparison to the outer edge, there is an increase of the plate mass next to the inner edge of the plate. As the result, next to the inner edge it can be noticed an area invulnerable to forced vibration due to significant concentration of the.

This phenomenon can be examined more precisely by analysing the influence of the rib width on the dynamic response of the system. On the Figure 4 the deflection of the plate at the time of $t = 2$ s in cases of different ribs width d is presented. It can be observed that the area insusceptible to forced vibration appears for ribs wider than $d = 0.01$m and is expanding with the increase of the width of the rib.

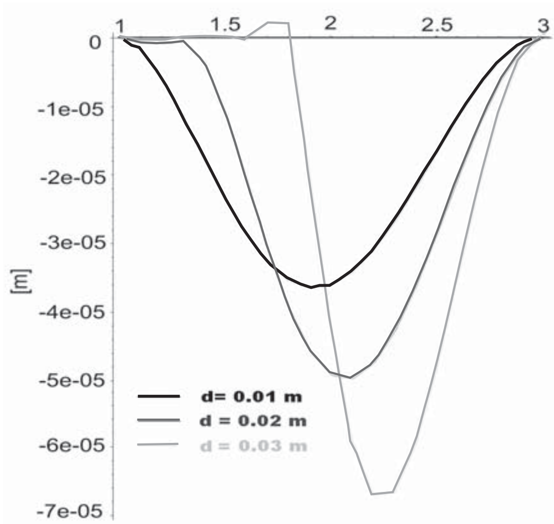

Fig. 4. The deflection of the plate for different width of the ribs at the time of $t = 2$ s. In all of the cases the rib is 10x denser, namely much less susceptible to forced vibrations than to matrix material

Rys. 4. Ugięcie płyty dla różnych szerokości żeber w chwili czasu $t = 2$ s. We wszystkich przypadkach żebro jest 10x gęstsze, a przez to znacznie mniej podatne na wzbudzenie drgań niż matryca

CONCLUSIONS

As the result of modelling and numerical calculation there can be drawn the conclusions:

- tolerance averaging technique allows for dynamic modelling of the behaviour of the plate made of functionally variable material properties,
- on the obtained diagrams it can be observed the area insusceptible to forced vibration as the influence of gradient effect of structural properties on the inner edge of annular plate that creates,

– stiffness modification of ribs/matrix has no influence on the shape of the plate response to forced vibration.

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DRGANIA WYMUSZONE Z TŁUMIENIEM PŁYTY PIERŚCIENIOWEJ WYKONANEJ Z MATERIAŁU O FUNKCYJNEJ GRADACJI WŁASNOŚCI

Streszczenie. Praca dotyczy zagadnienia dynamiki ciał wykonanych z inteligentnych materiałów, w których określona, deterministyczna budowa mikrostrukturalna ma przełożenie na makroskopowe własności całego ustroju konstrukcyjnego. W szczególności przedmiotem analizy jest płyta pierścieniowa wykonana z materiału o funkcyjnej gradacji makrowłasności (FGM). Struktura ta na poziomie mikrostrukturalnym jest zbudowana z dwóch materiałów: żeber o stałej szerokości oraz matrycy. W kierunku obwodowym struktura jest periodyczna. W kierunku promieniowym uśrednione własności struktury zmieniają się w sposób funkcyjnie zmienny. W pracy badane jest zagadnienie drgań wymuszonych z uwzględnieniem tłumienia. Punktem wyjścia rozważań jest równanie ruchu płyty cienkiej według teorii Kirchhoffa. Ostatecznie w wyniku procedury modelowania otrzymano układ równań cząstkowych różniczkowych na niewiadome, wolnozmienne funkcje. Układ ten posiada ciągłe współczynniki i może być rozwiązany metodami numerycznymi. W pracy pokazano także przykładowe rozwiązanie równań modelu dla zagadnienia asymptotycznego. Określono formę pierwszej, symetrycznej postaci drgań dla kilku przykładowych zestawów parametrów zadania, wybranych warunków początkowo-brzegowych oraz wymuszenia drgań.

Słowa kluczowe: metoda tolerancyjnego uśredniania, drgania płyt, FGM

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