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THE EFFECT OF DIFFERENCES OF MATERIAL PROPERTIES ON FREE VIBRATION FREQUENCIES OF SIMPLY SUPPORTED THIN TRANSVERSALLY GRADED PLATE BANDS

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Abstract. A certain analysis of free vibration frequencies of a plate band with a smooth and a slow gradation of properties on the macro-level is made in this article. These plate bands have a tolerance-periodic structure on the micro-level. Hence, it can be shown that for such objects the effect of the microstructure size plays a crucial role in dynamic problems, cf. Jędrysiak [2009], Kaźmierczak and Jędrysiak [2011]. In order to describe this effect the tolerance model of these bands is applied in this paper. Moreover to evaluate obtained results the asymptotic model is used. Fundamental free vibrations frequencies of the plate band, using the Ritz method are calculated using these models. Higher free vibrations frequencies are also obtained in the framework of the tolerance model. Moreover the effect of differences of Young's modulus and of mass densities in the cell on the microlevel is shown.

Key words: thin transversally graded plate band, the effect of the microstructure size, free vibration frequencies, the effect of distribution functions and differences of material properties

INTRODUCTION

Free vibrations of thin plate bands with a span L are investigated in this paper. The material macrostructure of these plate bands is functionally graded along their span (on the macrolevel). However, the microstructure of them is tolerance-periodic on the microlevel, cf. Jędrysiak [2010], Jędrysiak and Michalak [2011], Kaźmierczak and Jędrysiak [2010, 2011, 2013]. Thus, these plate bands can be called thin functionally graded plate bands, cf. Suresh and Mortensen [1998], Jędrysiak [2010]. The material properties of the plates are assumed to be independent of x_2-coordinate. A fragment of the plate band is shown in Fig. 1. The microstructure size is described by the length l of "the cell" and is assumed to be very small compared to span L of the plate.

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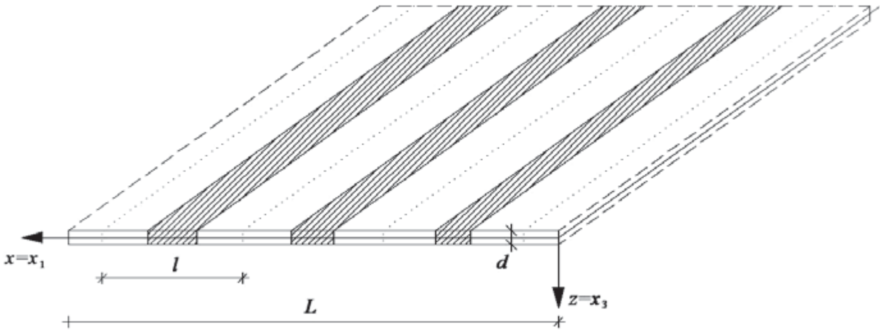

Fig. 1. A fragment of a thin transversally graded plate band
Rys. 1. Fragment cienkiego pasma płytowego o poprzecznej gradacji własności

Plates of this kind are described by partial differential equations with highly oscillating, tolerance-periodic, non-continuous coefficients, which are not a good tool to analyse vibrations of these plates. In order to make such analysis, various averaged models are formulated, which are determined by partial differential equations with smooth, slowly-varying coefficients. These plates can be treated as made of *a functionally graded material* [Suresh and Mortensen 1998], they are called *transversally graded plates* [Jędrysiak 2010].

Functionally graded structures are often described using approaches, which are applied to analyse macroscopically homogeneous media, e.g. periodic. Some of them are presented by Suresh and Mortensen [1998]. It should be mentioned these models, based on the asymptotic homogenization, cf. Jikov et al. [1994]. There are presented theoretical and numerical results of various problems of functionally graded structures in many papers. A collocation method with higher-order plate theories is used to analyse vibrations of FG-type plates by Roque et al. [2007]. A GDQ solution for free vibrations of shells is shown by Tornabene et al. [2011]. Higher order deformation theories are applied to investigate static response for functionally graded plates and shells by Oktem et al. [2012]. Shell-like structures with functionally graded material properties are investigated using a new low-order shell element by Kugler et al. [2013]. Free vibrations of functionally graded thick plates with shear and normal deformations effects are analysed by Jha et al. [2013]. An extended list of papers, where some theoretical and numerical results of thermomechanical problems of functionally graded structures can be found in Jędrysiak [2010] and Woźniak et al. (ed.) [2008, 2010]. Unfotunately, the governing equations of these models neglect usually the effect of the microstructure size.

In order to take into account this effect also in governing equations *the tolerance modelling* can be used [Woźniak et al. (ed.) 2008, 2010]. This method is applied to investigate various thermomechanical problems of periodic structures. Applications of the method can be found in a series of papers. Here, it can be mentioned those related to problems of periodic plates or shells, e.g. Michalak [2002], Nagórko and Woźniak [2002], Jędrysiak [2003, 2009], Jędrysiak and Paś [2005], Baron [2006], Tomczyk [2007, 2013], Domagalski and Jędrysiak [2012]. The tolerance modelling method is also adopted for similar thermomechanical problems of functionally graded structures, e.g. Jędrysiak [2010], Woźniak et al. (ed.) [2010]. Some applications to dynamic and stability problems for thin transversally graded plates are shown by: Kaźmierczak and Jędrysiak [2010, 2011, 2013], Jędrysiak and Michalak [2011], Jędrysiak [2013]; for functionally graded skeletonal shells by Michalak [2012]; for thin longitudinally graded plates by: Wirowski [2012], Michalak and Wirowski [2012]. The extended list of papers can be found in the books edited by Woźniak et al. (ed.) [2008, 2010].

The main aim of this paper is to apply the tolerance and the asymptotic models of vibrations for thin transversally graded plate bands to calculate free vibration frequencies of a simply supported plate band using the Ritz method. The second aim is to analyse the effect of various distribution functions of material properties on the frequencies. The third is to show the effect of differences between material properties [Young's modulus and mass densities] in the cell on the frequencies. These effects are investigated for simply supported thin transversally graded plate bands.

FORMULATION OF THE PROBLEM

Our considerations are treated as independent of x_2-coordinate. Denote $x = x_1$, $z = x_3$, $x \in [0, L]$ $\quad z \in [-d/2, d/2]$, with d as a constant plate thickness. Hence, the plate band is described in the interval $\Lambda = (0, L)$, with "the basic cell" $\Delta \equiv [-l/2, l/2]$ in the interval $\bar{\Lambda}$, where l is the length of the basic cell, satisfying conditions: $d << l << L$. Let a cell with a centre at $x \in \Lambda$ be denoted by $\Delta(x) \equiv (x - l/2, x + l/2)$. It is assumed that the plate band is made of two elastic isotropic materials, perfectly bonded across interfaces. These materials are characterised by Young's moduli E', E'', Poisson's ratios ν', ν'' and mass densities ρ', ρ''. Let us assume that $E(x), \rho(x), x \in \Lambda,$ are tolerance-periodic, highly-oscillating functions in x, but Poisson's ratio $\nu \equiv \nu' = \nu''$ is constant. Assuming $E' \neq E''$ and/or $\rho' \neq \rho''$ the plate material structure can be treated as transversally functionally graded in the x-axis direction. Let ∂ denote a derivative of x and $w(x, t)$ $(x \in \bar{\Lambda}, \quad t \in (t_0, t_1))$ be a plate band deflection.

Plate band properties are described by tolerance-periodic functions in x – the mass density per unit area of the midplane μ, the rotational inertia ϑ and the bending stiffness B:

$$\mu(x) \equiv d\rho(x), \quad \vartheta(x) \equiv \tfrac{d^3}{12}\rho(x), \quad B(x) \equiv \tfrac{d^3}{12(1-\nu^2)}E(x) \tag{1}$$

respectively. Free vibrations of thin transversally graded plate bands, under the assumptions of the Kirchhoff-type plate theory, are described by the partial differential equation of the fourth order for deflection $w(x, t)$:

$$\partial\partial[B(x)\partial\partial w(x,t)] + \mu(x)\ddot{w}(x,t) - \partial(\vartheta\partial\ddot{w}) = 0 \tag{2}$$

with highly-oscillating, non-continuous, tolerance-periodic functional in x coefficients.

FOUNDATIONS OF THE MODELLING

Following the book edited by Woźniak et al. (ed.) [2010] some of basic concepts of the tolerance modelling, which are also reformulated for tolerance-periodic plates by Jędrysiak [2010], are applied: *the tolerance-periodic function* $f \in TP_\delta^2(\Lambda, \Delta)$, *the slowly-varying function* $F \in SV_\delta^2(\Lambda, \Delta)$, *the highly-oscillating function* $\phi \in HO_\delta^\alpha(\Lambda, \Delta)$, *the fluctuation shape function* $FS_\delta^2(\Lambda, \Delta)$, with δ as a tolerance parameter, 2 as a supper indice determined a kind of that function, cf. also Kaźmierczak and Jędrysiak [2010].

The known averaging operator for an integrable function *f* is defined by:

$$< f > (x) = \frac{1}{l}\int_{\Delta(x)} f(y)dy, \quad x \in \Lambda_\Delta \tag{3}$$

where a cell at $x \in \Lambda_\Delta$ is denoted by $\Delta(x) \equiv x + \Delta, \quad \Lambda_\Delta = \{x \in \Lambda : \quad \Delta(x) \subset \Lambda\}$. If *f* is a tolerance-periodic function in *x* its averaged value calculated by (3) is a slowly-varying function in *x*.

Following the books by Woźniak et al. (ed.) [2010] and Jędrysiak [2010] and applying the basic concepts, the two fundamental modelling assumptions can be formulated.

The first assumption is *the micro-macro decomposition* of the plate band deflection *w*:

$$w(x,t) = W(x,t) + h^A(x)V^A(x,t), \quad A = 1, \ldots, N, \quad x \in \Lambda \tag{4}$$

with $W(\cdot,t), V^A(\cdot,t) \in SV_\delta^2(\Lambda, \Delta)$ (for every *t*) as basic kinematic unknowns ($W(\cdot, t)$ is called *the macrodeflection*; $V^A(\cdot, t)$ are called *the fluctuation amplitudes*), and $h^A(\cdot) \in FS_\delta^2(\Lambda, \Delta)$ being the known fluctuation shape functions.

The second modelling assumption is *the tolerance averaging approximation*, in which it is assumed that terms $O(\delta)$ are negligibly small in the course of modelling.

THE TOLERANCE MODELLING PROCEDURE

Following the monograph Woźniak et al. (ed.) [2010] the modelling procedure can be outlined in the form.

The formulation of the action functional is the first step:

$$\mathcal{A}(w(\cdot)) = \int_\Lambda \int_{t_0}^{t_1} \mathcal{L}(y, \partial\partial w(y,t), \partial\dot{w}(y,t), \dot{w}(y,t))\, dtdy \tag{5}$$

where lagrangean $\mathcal{L}$ is given by:

$$\mathcal{L} = \frac{1}{2}(\mu\dot{w}\dot{w} + \vartheta\partial\dot{w}\partial\dot{w} - B\partial\partial w\partial\partial w) \tag{6}$$

Using the principle stationary action, after some manipulations, the known equation (2) of free vibrations for thin transversally graded plate bands is derived.

In the next step of the tolerance modelling micro-macro decomposition (4) is substituted to (6). In the third step, applying averaging operator (3) the tolerance averaged form $<\mathcal{L}_h>$ of lagrangean (6) is obtained:

$$<\mathcal{L}_h> = -\tfrac{1}{2}\{(< B > \partial\partial W + 2 < B\partial\partial h^B > V^B)\partial\partial W + < \vartheta > \partial\dot{W}\partial\dot{W} - \\ - < \mu > \dot{W}\dot{W} + < B\partial\partial h^A \partial\partial h^B > V^A V^B + < \vartheta\partial h^A \partial h^B > \dot{V}^A \dot{V}^B - < \mu h^A h^B > \dot{V}^A \dot{V}^B\} \tag{7}$$

The principle stationary action applied to averaged functional $\mathcal{A}_h$ with lagrangean (7) leads to the system of governing equations with slowly-varying functional in *x* coefficients.

MODEL EQUATIONS

Tolerance model equations

From the principle stationary action applied to averaged functional with lagrangean (7), after some manipulations, the following system of equations for $W(\cdot,t)$ and $V^A(\cdot,t)$ is derived:

$$\partial\partial(< B > (x)\partial\partial W + < B\partial\partial h^B > (x)V^B) + < \mu > (x)\ddot{W} - < \vartheta > (x)\partial\partial\ddot{W} = 0 \\ < B\partial\partial h^A > (x)\partial\partial W + < B\partial\partial h^A \partial\partial h^B > (x)V^B + (\underline{< \mu h^A h^B > (x)} + \underline{< \vartheta\partial_\alpha h^A \partial_\alpha h^B > (x)})\ddot{V}^B = 0 \tag{8}$$

The underlined terms in these equations depend on the microstructure parameter *l*. Coefficients of equations (8) are slowly-varying functions in *x*. These quations constitute the tolerance model of thin transversally graded plate bands, which allows to take into account the effect of the microstructure size on free vibrations of these plates. It can be observed that boundary conditions for these plate bands (in $\Lambda = (0,L)$) are formulated only for *macrodeflection W* (on edges $x = 0$, L), but not for *fluctuation amplitudes* V^A, $A = 1, \ldots, N$.

Asymptotic model equations

Neglecting terms with *l* in equations $(8)_2$ the algebraic equations for fluctuation amplitudes V^A are obtained:

$$V^A = -(< B\partial\partial h^A \partial\partial h^B >)^{-1} < B\partial\partial h^B > \partial\partial W \tag{9}$$

Substituting formula (9) into $(8)_1$ we arrive at the following equation for $W(\cdot,t)$:

$$\partial\partial((< B > (x) - < B\partial\partial h^A > (x)(< B\partial\partial h^A \partial\partial h^B > (x))^{-1} < B\partial\partial h^B > (x))\partial\partial W) + < \mu > (x)\ddot{W} = 0 \tag{10}$$

The asymptotic model of thin transversally graded plate bands is represented by the above equation and micro-macro decomposition (4). This model can be obtained in the framework of the formal asymptotic modelling procedure, cf. the book by Woźniak et al. (ed.) [2010], Kaźmierczak and Jędrysiak [2011, 2013]. The effect of the microstructure size on free vibrations of the transversally graded plates is neglected in equation (10). The asymptotic model describes only the macrobehaviour of these plate bands.

AN ANALYSIS OF FREE VIBRATIONS OF PLATE BANDS

Introduction

Let us consider free vibrations of a simply supported thin plate band with span L along the x-axis. The properties of the plate band are described by the following functions:

$$\rho(\cdot, z), E(\cdot, z) = \begin{cases} \rho', E', & \text{for} \quad z \in ((1-\gamma(x))l/2, (1+\gamma(x))l/2) \\ \rho'', E'', & \text{for} \quad z \in [0, (1-\gamma(x))l/2] \cup [(1+\gamma(x))l/2, l], \end{cases} \tag{11}$$

where $\gamma(x)$ is a distribution function of material properties, cf. Fig. 2.

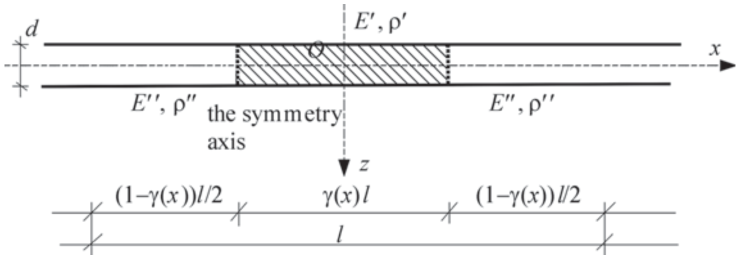

Fig. 2. “Basic cell” of the transversally graded plate band
Rys. 2. „Komórka podstawowa” pasma płytowego o poprzecznej gradacji własności

Let us assume only one fluctuation shape function, i.e. $A = N = 1$, and denote $h \equiv h^1$, $V \equiv V^1$. Hence, micro-macro decomposition (4) of field $w(x, t)$ can be written in the form:

$$w(x,t) = W(x,t) + h(x)V(x,t),$$

where $W(\cdot,t),\ V(\cdot,t) \in SV_\delta^2(\Lambda, \Delta)$ for every $t \in (t_0, t_1),\ h(\cdot) \in FS_\delta^2(\Lambda, \Delta)$.

The cell has a structure shown in Fig. 2. Thus, the periodic approximation of the fluctuation shape function $h(x)$ takes the form:

$$\tilde{h}(x,z) = \lambda^2[\cos(2\pi z/l) + c(x)], \quad z \in \Delta(x), \quad x \in \Lambda,$$

where parameter $c(x)$ is a slowly-varying function in x and is determined by $<\tilde{\mu}\tilde{h}>=0$:

$$c = c(x) = \frac{\sin[\pi\tilde{\gamma}(x)](\rho' - \rho'')}{\pi\{\rho'\tilde{\gamma}(x) + \rho''[1-\tilde{\gamma}(x)]\}},$$

with $\tilde{\gamma}(x)$ as the periodic approximation of the distribution function of material properties $\gamma(x)$. Parameter $c(x)$ is treated as constant in calculations of derivatives $\partial\tilde{h}$, $\partial\partial\tilde{h}$.

Denote:

$$\begin{aligned} &\breve{B} \equiv <B>, \quad \widehat{B} \equiv <B\partial\partial h>, \quad \overline{B} = <B\partial\partial h\partial\partial h>, \\ &\breve{\mu} = <\mu>, \quad \overline{\mu} = l^{-4} <\mu hh>, \quad \breve{\vartheta} = <\vartheta>, \quad \overline{\vartheta} = l^{-2} <\vartheta\partial h\partial h> \end{aligned} \tag{12}$$

Hence, tolerance model equations (8) take the form:

$$\begin{aligned} &\partial\partial(\breve{B}\partial\partial W + \widehat{B}V) + \breve{\mu}\ddot{W} - \breve{\vartheta}\partial\partial\ddot{W} = 0 \\ &\widehat{B}\partial\partial W + \overline{B}V + l^2(l^2\overline{\mu} + \overline{\vartheta})\ddot{V} = 0 \end{aligned} \tag{13}$$

Moreover, using denotations (12), the plate band equation (10) has the form:

$$\partial\partial[(\breve{B} - \widehat{B}^2 / \overline{B})\partial\partial W] + \breve{\mu}\ddot{W} - \breve{\vartheta}\partial\partial\ddot{W} = 0 \tag{14}$$

Equation (14) describes free vibrations of this plate band within the asymptotic model. All coefficients of equations (13) and (14) are slowly-varying functions in x.

The Ritz method applied to the model equations

Since, analytical solutions of equations (13) or (14), with slowly-varying, functional coefficients, are too difficult to solve, approximate formulas of free vibrations frequencies will be derived using the known Ritz method, cf. Kaźmierczak and Jędrysiak [2010]. Hence, relations of the maximal strain energy $\mathcal{U}_{\max}$ and the maximal kinetic energy $\mathcal{K}_{\max}$ are determined.

Solutions to equation (14) and equations (13) are assumed in the form satisfying boundary conditions for the simply supported plate band:

$$W(x,t) = A_W \sin(\alpha x)\cos(\omega t), \quad V(x,t) = A_V \sin(\alpha x)\cos(\omega t) \tag{15}$$

with a wave number α and a free vibrations frequency ω. Introducing denotations:

$$\breve{B} = \tfrac{d^3}{12(1-\nu^2)} \int_0^L \{E''[1-\tilde{\gamma}(x)] + \tilde{\gamma}(x)E'\}[\sin(\alpha x)]^2\, dx,$$

$$\overline{B} = \tfrac{(\pi d)^3}{3(1-\nu^2)} \int_0^L \{(E' - E'')[2\pi\tilde{\gamma}(x) + \sin(2\pi\tilde{\gamma}(x))] + 2\pi E''\}[\sin(\alpha x)]^2\, dx,$$

$$\widehat{B}=\tfrac{\pi d^3}{3(1-\nu^2)}(E'-E'')\int_0^L \sin(\pi\tilde{\gamma}(x))[\sin(\alpha x)]^2\,dx,$$

$$\breve{\mu}=d\int_0^L \{[1-\tilde{\gamma}(x)]\rho''+\tilde{\gamma}(x)\rho'\}[\sin(\alpha x)]^2\,dx,$$

$$\breve{\vartheta}=\tfrac{d^3}{12}\int_0^L \{[1-\tilde{\gamma}(x)]\rho''+\tilde{\gamma}(x)\rho'\}[\cos(\alpha x)]^2\,dx,$$

$$\begin{aligned}\overline{\mu}=\tfrac{d}{4\pi}\int_0^L &\{(\rho'-\rho'')[2\pi\tilde{\gamma}(x)+\sin(2\pi\tilde{\gamma}(x))]+2\pi\rho''\}[\sin(\alpha x)]^2\,dx+\\ &+\tfrac{d}{\pi}(\rho'-\rho'')\int_0^L c(x)[\pi c(x)\tilde{\gamma}(x)-2\sin(\pi\tilde{\gamma}(x))][\sin(\alpha x)]^2\,dx+\\ &+d\rho''\int_0^L [c(x)]^2[\sin(\alpha x)]^2\,dx,\end{aligned} \tag{16}$$

$$\overline{\vartheta}=\tfrac{\pi d^3}{12}\int_0^L \{(\rho'-\rho'')[2\pi\tilde{\gamma}(x)-\sin(2\pi\tilde{\gamma}(x))]+2\pi\rho''\}[\Psi(\alpha x)]^2\,dx$$

and using (15) for the tolerance model formulas of the maximal energies – strain $\mathcal{U}_{\max}$ and kinetic $\mathcal{K}_{\max}$ take the form:

$$\begin{aligned}\mathcal{U}^{AM}_{\max}&=\tfrac{1}{2}[(\breve{B}A_W{}^2\alpha^2-2\widehat{B}A_W A_V)\alpha^2+\overline{B}A_V{}^2]\\ \mathcal{K}^{AM}_{\max}&=\tfrac{1}{2}[A_W{}^2(\breve{\mu}+\breve{\vartheta}\alpha^2)+A_V{}^2 l^2(l^2\overline{\mu}+\overline{\vartheta})]\omega^2\end{aligned} \tag{17}$$

However for the asymptotic model they have the form:

$$\mathcal{U}^{AM}_{\max}=\tfrac{1}{2}[(\breve{B}A_W{}^2\alpha^2-2\widehat{B}A_W A_V)\alpha^2+\overline{B}A_V{}^2],\qquad \mathcal{K}^{AM}_{\max}=\tfrac{1}{2}A_W{}^2(\breve{\mu}+\breve{\vartheta}\alpha^2)\omega^2 \tag{18}$$

Using the conditions of the Ritz method:

$$\frac{\delta(\mathcal{U}_{\max}-\mathcal{K}_{\max})}{\delta A_W}=0,\qquad \frac{\delta(\mathcal{U}_{\max}-\mathcal{K}_{\max})}{\delta A_V}=0 \tag{19}$$

from relations (17) after some manipulations the following formulas are obtained:

$$\begin{aligned}(\omega_{-,+})^2\equiv&\frac{\alpha^4 l^2(l^2\overline{\mu}+\overline{\vartheta})\breve{B}+(\breve{\mu}+\alpha^2\breve{\vartheta})\overline{B}}{2(\breve{\mu}+\alpha^2\breve{\vartheta})l^2(l^2\overline{\mu}+\overline{\vartheta})}\mp\\ &\mp\frac{\sqrt{[\breve{B}\alpha^4 l^2(l^2\overline{\mu}+\overline{\vartheta})-(\breve{\mu}+\alpha^2\breve{\vartheta})\overline{B}]^2+4(\alpha^2 l\widehat{B})^2(\breve{\mu}+\alpha^2\breve{\vartheta})(l^2\overline{\mu}+\overline{\vartheta})}}{2(\breve{\mu}+\alpha^2\breve{\vartheta})l^2(l^2\overline{\mu}+\overline{\vartheta})}\end{aligned} \tag{20}$$

for the lower ω_- and the higher ω_+ free vibrations frequencies, respectively, in the framework of the tolerance model.

For the asymptotic model conditions (19) are applied to equations (18) and after manipulations we arrive at the following formula:

$$\omega^2 \equiv \alpha^4 \frac{\breve{B}\overline{B} - \widehat{B}^2}{(\breve{\mu} + \breve{\vartheta}\alpha^2)\overline{B}}, \tag{21}$$

of the lower free vibrations frequency ω.

Results

Let us introduce the distribution functions of material properties $\gamma(x)$ in the following form:

$$\tilde{\gamma}(x) = \sin^2(\pi x / L) \tag{22}$$

$$\tilde{\gamma}(x) = \cos^2(\pi x / L) \tag{23}$$

$$\tilde{\gamma}(x) = (x / L)^2 \tag{24}$$

$$\tilde{\gamma}(x) = \sin(\pi x / L) \tag{25}$$

$$\tilde{\gamma}(x) = 0.5 \tag{26}$$

where formula (26) determines an example of a periodic plate band.

Moreover, let us denote by:

$$\Omega^2 \equiv \tfrac{12(1-\nu^2)\rho'}{E'} l^2 \omega^2, \qquad (\Omega_-)^2 \equiv \tfrac{12(1-\nu^2)\rho'}{E'} l^2 (\omega_-)^2, \qquad (\Omega_+)^2 \equiv \tfrac{12(1-\nu^2)\rho'}{E'} l^2 (\omega_+)^2 \tag{27}$$

dimensionless frequency parameters for the free vibrations frequencies ω and ω_-, ω_- determined by equations (21) and (20), respectively.

Results of calculations are shown in Fig. 3 and 4, where there are presented results obtained by the tolerance or the asymptotic models for plate bands with the simply supported edges. Fig. 3 shows plots of the lower frequency parameters versus both ratios $E''/E' - \rho''/\rho'$, but Fig. 4 shows diagrams of the higher frequency parameters versus these both ratios. These calculations are made for the Poisson's ratio $\nu = 0.3$, the wave number $\alpha = \pi/L$, ratio $l/L = 0.1$ and ratio $d/l = 0.1$.

Some remarks can be formulated from results presented in Fig. 3 and 4:

1. The effect of distribution functions of material properties $\gamma(x)$ on the lower frequency parameters for various ratios $E''/E' \in [0;1]$, $\rho''/\rho' \in [0;1]$ can be observed in Fig. 3:

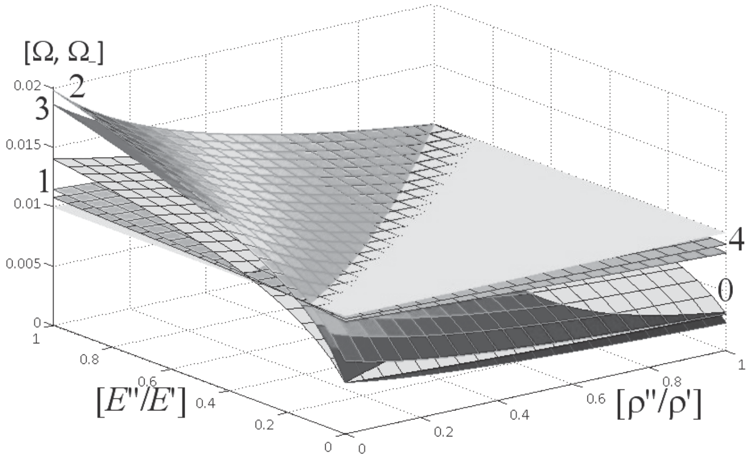

Fig. 3. Plots of dimensionless frequency parameters Ω and Ω_- of lower free vibration frequencies versus ratios $E''/E' - \rho''/\rho'$, for the simply supported plate band (1 – γ by (22); 2 – γ by (23); 3 – γ by (24); 4 – γ by (25); 0 – γ by (26); a grey plane is related to the frequency parameter for the homogeneous plate band, i.e. $E''/E' = \rho''/\rho' = 1$)

Rys. 3. Wykresy bezwymiarowych parametrów częstości Ω i Ω_- niższych częstości drgań swobodnych, w zależności od ilorazów $E''/E' - \rho''/\rho'$, dla przegubowo podpartego pasma płytowego (1 – γ wg (22); 2 – γ wg (23); 3 – γ wg (24); 4 – γ wg (25); 0 – γ wg (26); szara płaszczyzna stanowi wykres parametru częstości jednorodnego pasma płytowego, tj. $E''/E' = \rho''/\rho' = 1$)

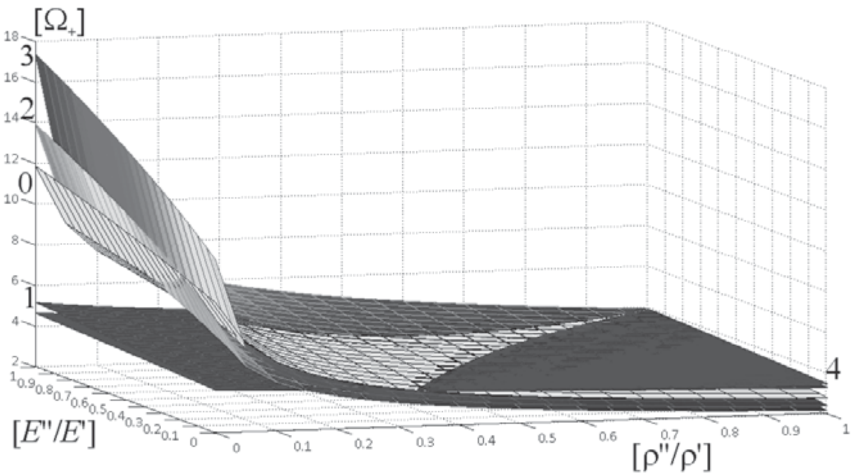

Fig. 4. Plots of dimensionless frequency parameters Ω_+ of higher free vibration frequencies versus ratios $E''/E' - \rho''/\rho'$, for the simply supported plate band (1 – γ by (22); 2 – γ by (23); 3 – γ by (24); 4 – γ by (25); 0 – γ by (26))

Rys. 4. Wykresy bezwymiarowych parametrów częstości Ω_+ wyższych częstości drgań swobodnych, w zależności od ilorazów $E''/E' - \rho''/\rho'$, dla przegubowo podpartego pasma płytowego (1 – γ wg (22); 2 – γ wg (23); 3 – γ wg (24); 4 – γ wg (25); 0 – γ wg (26))

- the highest values of these frequency parameters for the simply supported plate band, cf. Fig. 3, are obtained:
  - for $\gamma(x)$ by (23) and for pairs of ratios $(E''/E', \rho''/\rho')$, such that $E''/E' > (E''/E')_0 > 0$, $\rho''/\rho' < (\rho''/\rho')_0((E''/E')_0) > 0$, where $(\rho''/\rho')_0$ depends on $(E''/E')_0$),
  - for $\gamma(x)$ by (25) and for pairs of ratios $(E''/E', \rho''/\rho')$, such that $E''/E' < (E''/E')_0 > 0$, $\rho''/\rho' > (\rho''/\rho')_0((E''/E')_0) > 0$, where $(\rho''/\rho')_0$ depends on $(E''/E')_0$;
- the smallest values of these frequency parameters, cf. Fig. 3, are obtained:
  - for $\gamma(x)$ by (25) and for pairs of ratios $(E''/E', \rho''/\rho')$, such that $E''/E' > (E''/E')_1 > 0$, $\rho''/\rho' < (\rho''/\rho')_1((E''/E')_1) > 0$, where $(\rho''/\rho')_1$ depends on $(E''/E')_1$),
  - for $\gamma(x)$ by (24) and for pairs of ratios $(E''/E', \rho''/\rho')$, such that $E''/E' < (E''/E')_2 > 0$, $\rho''/\rho' > (\rho''/\rho')_2((E''/E')_2) > 0$, where $(\rho''/\rho')_2$ depends on $(E''/E')_2$),
  - moreover, for $\gamma(x)$ by (26) (the periodic plate band) and for pairs of ratios $(E''/E', \rho''/\rho')$, such that $(E''/E')_1 > E''/E' > (E''/E')_2 > 0$, $(\rho''/\rho')_1((E''/E')_1) < \rho''/\rho' < (\rho''/\rho')_2((E''/E')_2) > 0$, where $(\rho''/\rho')_1$, $(\rho''/\rho')_2$ depend on $(E''/E')_1$, $(E''/E')_2$), respectively;

2. Fig. 3 shows also an interesting feature that for the used distribution functions of material properties $\gamma(x)$ the lower frequency parameters are:

- higher than the lower frequency parameter for the homogeneous plate band with ratios $E''/E' = \rho''/\rho' = 1$ (a grey plane in Fig. 3) for pairs of ratios $(E''/E', \rho''/\rho')$, such that $E''/E' > (E''/E')_1 > 0$, $\rho''/\rho' < (\rho''/\rho')_1((E''/E')_1) > 0$, (and $(\rho''/\rho')_1$ depends on $(E''/E')_1$);
- smaller than the lower frequency parameter for the homogeneous plate band with ratios $E''/E' = \rho''/\rho'=1$ (a grey plane in Fig. 3) for pairs of ratios $(E''/E', \rho''/\rho')$, such that $E''/E' < (E''/E')_0 > 0$, $\rho''/\rho' > (\rho''/\rho')_0((E''/E')_0) > 0$, (and $(\rho''/\rho')_0$ depends on $(E''/E')_0$).

3. The effect of distribution functions of material properties $\gamma(x)$ on the higher frequency parameters for various ratios $E''/E' \in [0;1]$, $\rho''/\rho' \in [0;1]$ can be observed in Fig. 4:

- the highest values of the higher frequency parameters for the simply supported plate band, cf. Fig. 4, are obtained:
  - for $\gamma(x)$ by (24) and for pairs of ratios $(E''/E', \rho''/\rho')$, such that $E''/E' > (E''/E')_0 > 0$, $\rho''/\rho' < (\rho''/\rho')_0((E''/E')_0) > 0$, where $(\rho''/\rho')_0$ depends on $(E''/E')_0$),
  - for $\gamma(x)$ by (25) and for pairs of ratios $(E''/E', \rho''/\rho')$, such that $E''/E' < (E''/E')_1 > 0$, $\rho''/\rho' > (\rho''/\rho')_1((E''/E')_1) > 0$, where $(\rho''/\rho')_1$ depends on $(E''/E')_1$,
  - moreover, for $\gamma(x)$ by (26) (the periodic plate band) for certain pairs of ratios $(E''/E', \rho''/\rho')$, such that $(E''/E')_2 > E''/E' > (E''/E')_0 > 0$ and $E''/E' > (E''/E')_1 > 0$, $\rho''/\rho' < (\rho''/\rho')_2((E''/E')_2) > 0$, where $(\rho''/\rho')_0$, $(\rho''/\rho')_1$, $(\rho''/\rho')_2$ depend on $(E''/E')_0$, $(E''/E')_1$, $(E''/E')_2$), respectively,
  - moreover, for $\gamma(x)$ by (23) for pairs of ratios $(E''/E', \rho''/\rho')$, such that $E''/E' > (E''/E')_2 > 0$, $\rho''/\rho' < (\rho''/\rho')_2((E''/E')_2) > 0$, where $(\rho''/\rho')_2$ depends on $(E''/E')_2$);

- the smallest values of the higher frequency parameters, cf. Fig. 4, are obtained:
  - for $\gamma(x)$ by (25) and for pairs of ratios (E''/E', ρ''/ρ'), such that $E''/E' > (E''/E')_3 > 0$, $\rho''/\rho' < (\rho''/\rho')_3((E''/E')_3) > 0$, where $(\rho''/\rho')_3$ depends on $(E''/E')_3$),
  - for $\gamma(x)$ by (23) and for pairs of ratios (E''/E', ρ''/ρ'), such that $(E''/E')_3 > E''/E' > 0$, $\rho''/\rho' > (\rho''/\rho')_3((E''/E')_3) > 0$, where $(\rho''/\rho')_3$ depends on $(E''/E')_3$).

REMARKS

The averaged tolerance model equations of the functionally graded plate bands are derived using the tolerance modelling to the known differential equation of Kirchhoff-type plates. This method leads from the differential equation with non-continuous, tolerance-periodic coefficients to the system of differential equations with slowly-varying coefficients. The tolerance model equations involve terms describing the effect of the microstructure size on the overall behaviour of these plates. But the asymptotic model describes only their macrobehaviour.

In the example free vibration frequencies of the simply supported plate band have been analysed for various distribution functions of material properties $\gamma(x)$ and different ratios of material properties E''/E', ρ''/ρ'.

Analysing results of this example it can be observed that:

1. Lower free vibrations frequencies can be analysed using both the presented models – the tolerance and the asymptotic.

2. Lower and higher free vibrations frequencies decrease with the increasing of ratio ρ''/ρ', but they increase with the increasing of ratio E''/E'.

3. Using various distribution functions of material properties $\gamma(x)$ there can be made microstructured plates having lower fundamental free vibrations frequencies smaller or higher than these frequencies for the homogeneous plate made of the stronger material (i.e. the plate with ratios $E''/E' = \rho''/\rho' = 1$) for different pairs of ratios (E''/E', ρ''/ρ').

Other problems of vibrations for the functionally graded plates under consideration and some evaluations of obtained results will be shown in forthcoming papers.

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WPŁYW RÓŻNIC WŁASNOŚCI MATERIAŁOWYCH NA CZĘSTOŚCI DRGAŃ WŁASNYCH SWOBODNIE PODPARTYCH CIENKICH PASM PŁYTOWYCH O POPRZECZNEJ GRADACJI WŁASNOŚCI

Streszczenie. W tej pracy pokazano pewną analizę częstości drgań swobodnych pasma płytowego o gładkiej i wolnej zmianie własności na poziomie makro. Takie pasma płytowe mają budowę tolerancyjnie-periodyczną na poziomie mikro. Można więc wykazać, że w zagadnieniach dynamicznych takich obiektów wielkość mikrostruktury ma duże znaczenie [Jędrysiak 2009, Kaźmierczak i Jędrysiak 2011]. W celu opisania tego efektu zastosowano model tolerancyjnych tego rodzaju pasm płytowych. Ponadto otrzymane wyniki porównano z wynikami uzyskanymi modelem asymptotycznym. Podstawowe częstości drgań swobodnych pasma płytowego obliczono w obu modelach, korzystając z metody Ritza. Czeęstości wyższe otrzymano także w modelu tolerancyjnym. Pokazano również wpływ różnic modułów Younga i gęstości masy w komórce na poziomie mikro.

Słowa kluczowe: cienkie pasmo płytowe o poprzecznej gradacji własności, wpływ wielkości mikrostruktury, częstości drgań swobodnych, wpływ funkcji rozkładu własności i różnic materiałowych

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