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STRESS-DEFORMATION STATE OF COMPOSITE STEELCONCRETE CONSTRUCTIONS AT RESILIENT AXIS POSITION CHANGE

Ivan Ivanyk, Svitlana Vihot
Lviv Polytechnick National University, Lviv

Abstract. Calculation of combined steel-concrete structures shall be conducted in stages. Before monolithic slab concrete gains strength, metal structure is the only structure part that carries loading of its own weight. However, after concrete gains strength, the whole calculated loading is carried by whole reinforced monolithic concrete slab where axis position changes. The article describes main principles of the technique of imaginary hinges installation for the forces calculation in static indefinite continuous structures under conditions of mathematical axis position change in stiffening girder. That enables to simulate the stress-deformation state in the combined structures elements at the initial design stage. This technique is used for the calculation and results analysis for combined structure with composite reinforced concrete top band and steel suspension taking into account of their work stages.

Key words: resilient axis, composite constructions, equation of strain continuity

AIM OF WORK

The aim of the stress state regulation of building constructions is an improvement of design and technological processes, their assembling and work under loading. The problem of the equal stress state regulation in the constructions elements at the design stage consists in the use of calculation methodology. One of the task decision directions is the use of the method that was worked out and approved on the flat cross-ribbed systems [Ivanyk et al. 2006].

Corresponding author – Adres do korespondencji: Ivan Ivanyk, Svitlana Vihot, Narodowy Uniwersytet Politechnika Lwowska, Katedra Technologii Budownictwa, ul. Karpińskiego 6, 79013 Lwów, e-mail: ndl19@meta.ua

PROBLEM STATEMENT

During calculations of combined continued slab-beam systems, most often, they are replaced with simplified discrete physical models as the static indefinite system, the elements of that present the geometrical axes of beam and elements of subdiagonal, with equal stiffness.

Will consider a steel-concrete construction, that consists of $n = 2$ the longitudinal steel static indefinite combined constructions that are incorporated in compatible spatial work by means of monolithic reinforce-concrete slab (Fig. 1).

Fig. 1. Spatial combined static indefinite combined steel-concrete construction

Rys. 1. Przestrzenna statycznie niewyznaczalna konstrukcja zespolona stalowo-betonowa

The combined steel static indefinite construction that consists of stiffness beam and truss elements, uniformly loaded with loading of *q*. A loading scheme of construction is shown on Figure 2. As a result of beam deformation under loading, there are not only efforts in a static indefinite construction but also resilient axis position of beam and accordingly all knots of truss (Fig. 3) changes.

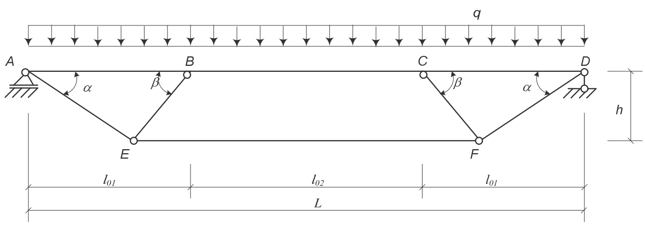

Fig. 2. Type of the combined static indefinite construction

Rys. 2. Szkieletowa statycznie niewyznaczalna konstrukcja

CALCULATION METHODOLOGY

For the further system salvation will use methodology of imaginary hinges introduction [Kvasha et al. 1997] for system characteristic cross-sections in the places of possible maximal moments in middle parts of spans at l_{01} and l_{02}. The calculation scheme of such construction will look like, represented on Figure 3.

The system calculation scheme, at introduction to the upper band imaginary hinges, is modelled as a construction in which in upper band as basic efforts are unknown bending

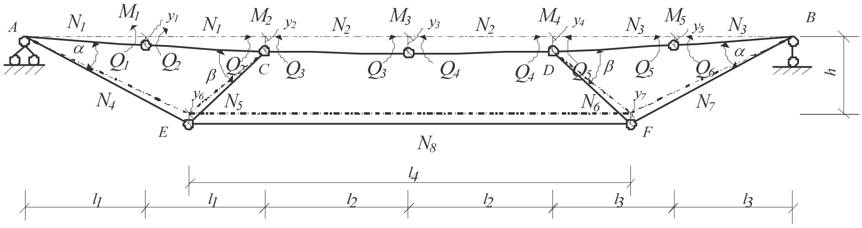

Fig. 3. The combined construction calculation scheme

Rys. 3. Schemat obliczeniowy konstrukcji

moments, transversal and longitudinal forces. There are only longitudinal efforts (Fig. 3) in the underbody of truss.

The resilient axis position changes as a result of deformation of the combined static indefinite construction under loading. Let us designate through $n = 0, ..., i$ amount of characteristic cross sections of beam, in that the external loading is applied or stiffness characteristics change.

The offered methodology is based on substituting of actual resilient axis of beam by fictitious axis with introduction of imaginary hinges with the simultaneous applying of bending moments in characteristic points. Those are the points of appliance of external forces or cross sections of stiffness change in the real beam. A resilient bended axis will represent as an axis from $n = 0, ..., i$ amount of intermediate hinges that as a result of deformation was displaced in vertical direction accordingly on the size of y_{np} ($n = 0, ..., i$). The occurrence of bending moments M_n, transverse forces Q_m ($m = 1, ..., i$) longitudinal forces N_m take as a result of displacement y_n points (nodes) of the resilient axis of the beam (Fig. 4).

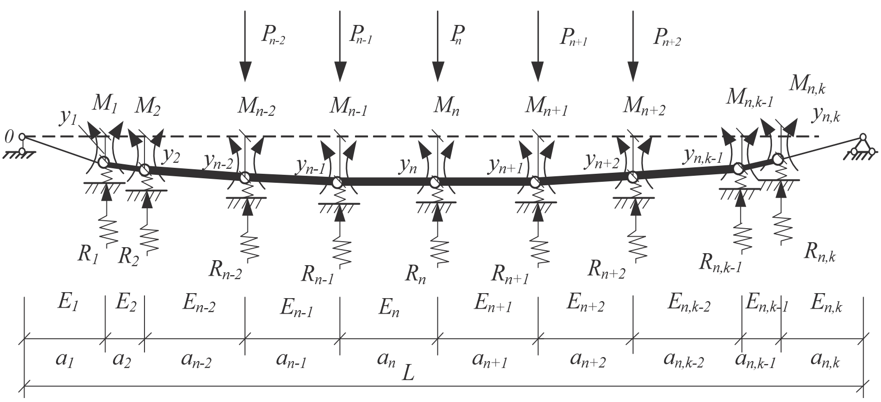

Fig. 4. Calculation scheme of the beam with variable stiffness in the deformed state

Rys. 4. Schemat obliczeniowy belki o zmiennej sztywności w stanie deformacji

Angle of rotation of relative cross sections of the *n*-th imaginary hinge that caused by bending moments M_{n-1}, M_n, M_{n+1} is written as:

$$\Delta_{nM} = M_{n-1} \cdot \delta_{n,n-1} + M_n \cdot \delta_{n,n} + M_{n+1} \cdot \delta_{n,n+1} \tag{1}$$

The equations of deformation continuity (equations of *i* effort), reflecting the interdependence of unknown bending moments along the length of stiffness beam, unknown longitudinal forces in the construction elements and vertical displacements of all nodes of combined construction will look like:

$$
\begin{aligned}
&\delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 + \delta_{14}X_4 + \delta_{15}X_5 + \delta_{16}X_6 + \delta_{17}X_7 - \frac{2y_1}{l_1} + \frac{y_2}{l_2} = 0 \\
&\delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 + \delta_{24}X_4 + \delta_{25}X_5 + \delta_{26}X_6 + \delta_{27}X_7 + \frac{y_1}{l_1} - \frac{y_2\left(l_1 + l_2\right)}{l_1 * l_2} + \frac{y_3}{l_2} = 0 \\
&\delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 + \delta_{34}X_4 + \delta_{35}X_5 + \delta_{36}X_6 + \delta_{37}X_7 + \frac{y_2}{l_2} - \frac{2y_3}{l_2} + \frac{y_4}{l_2} = 0 \\
&\delta_{41}X_1 + \delta_{42}X_2 + \delta_{43}X_3 + \delta_{44}X_4 + \delta_{45}X_5 + \delta_{46}X_6 + \delta_{47}X_7 + \frac{y_3}{l_2} - \frac{y_4\left(l_1 + l_2\right)}{l_1 * l_2} + \frac{y_5}{l_1} = 0 \\
&\delta_{51}X_1 + \delta_{52}X_2 + \delta_{53}X_3 + \delta_{54}X_4 + \delta_{55}X_5 + \delta_{56}X_6 + \delta_{57}X_7 + \frac{y_4}{l_1} - \frac{2y_5}{l_1} = 0 \\
&\delta_{61}X_1 + \delta_{62}X_2 + \delta_{63}X_3 + \delta_{64}X_4 + \delta_{65}X_5 + \delta_{66}X_6 + \delta_{67}X_7 + y_6 = 0 \\
&\delta_{71}X_1 + \delta_{72}X_2 + \delta_{73}X_3 + \delta_{74}X_4 + \delta_{75}X_5 + \delta_{76}X_6 + \delta_{77}X_7 + y_7 = 0
\end{aligned} \tag{2}
$$

Coefficients δ_{ij} of the equations unknowns of *i* effort (2) can be written in the form, for example:

$$
\begin{aligned}
\delta_{11} = \; & \frac{2l'_1}{3EI_{red1}} - \mu * \frac{2}{l_1 GA_{red1}} + \frac{2\overline{N}'^2_{11} * l'_1}{6EA_{red1}} + \frac{2\overline{N}'^2_{12} * l'_2}{6EA_{red2}} + \frac{2\overline{N}'^2_{12} * l'_1}{6EA_{red3}} \\
& + \frac{\overline{N}'^2_{14} * h'}{6\sin aEA_4} + \frac{\overline{N}'^2_{15} * h'}{6\sin \beta EA_5} + \frac{\overline{N}'^2_{16} * h'}{6\sin \beta EA_6} + \frac{\overline{N}'^2_{17} * h'}{6\sin aEA_7} + \frac{\overline{N}'^2_{18} * \left(l - 2h' * \mathrm{ctg}a\right)}{6EA_8}
\end{aligned} \tag{3}
$$

where: $\overline{N}'_{ij}$ – the values of longitudinal forces caused by action of the single moments in the nodes of the combined structure beam with a modified position of neutral axis,

$l'_{1,\, l'2}$ – values of outmost and mid spans of stiffness beam,

h' – mathematical height combined steel-concrete structures,

EI_{red1}, GA_{red1}, EA_{red1}, EA_{red2} – values of stiffness parameters of stiffness beam spans.

It is necessary to the continuity equations (2), which are insufficient for finding unknown X_i and y_i add the static equations, which are found from the equilibrium panels and nodes of combined trussing construction.

The equations of continuity and static deformations form a linear algebraic equations system, it is sufficient to find the unknown bending moments M_{i1} ($i_1 = 1, \ldots, i$), deflections y_{i2} ($i_2 = 1, \ldots, j$) and axial longitudinal forces N_{i3} ($i_3 = 1, \ldots, k$). When constructing steel concrete statically indeterminate structure, shown in Figure 1, there is displacement of the neutral axis of the stiffness beam due to changes in the dimensions of the reduced cross section – from steel stiffness beam to steel-concrete beam (Fig. 5).

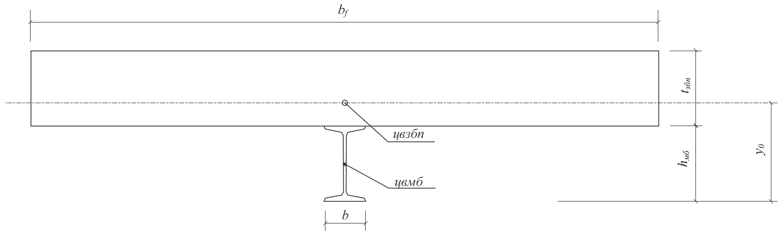

Fig. 5. Steel-concrete combined structure cross-section

Rys. 5. Przekrój poprzeczny konstrukcji zespolonej stalowo-żelbetowej

The stiffness parameters change of the beam $B(x)$ results in the topology change of statically indefinite combined steel-concrete structure, namely (Fig. 6 and 7): there is the vertical displacement of the stiffness beam mathematical axis from position of A–B in position of A'–B'; there is corresponding displacement of intersections of axes of action of central forces of elements of sub diagonal with new neutral axis – from points A, B, C, D in points A', B', C', D'; the sizes of extreme change – from l_1 to $l'_1 = l_1 + \Delta y_0/\text{tg}\alpha + \Delta y_0/\Delta\beta$ – (increase) and intermediate from l_2 to $l'_2 = l_2 - 2\Delta y_0/\text{tg}\beta$ – (diminish) spans of stiffness beam; the construction height increases: from h to $h' = h + \Delta y_0$. At moving of mathematical axis to the size Δy_{0i} (Fig. 6) there is already on the stage of arranging of monolithic reinforce-concrete slab a negative bending moment of M_A, caused by the action of N_{4y} (vertical constituent of central force N_4) in outmost sub-diagonal on horizontal relocation bias with the intersection of mathematical axis in the stiffness beam $\Delta y_0/\text{tg}\alpha$.

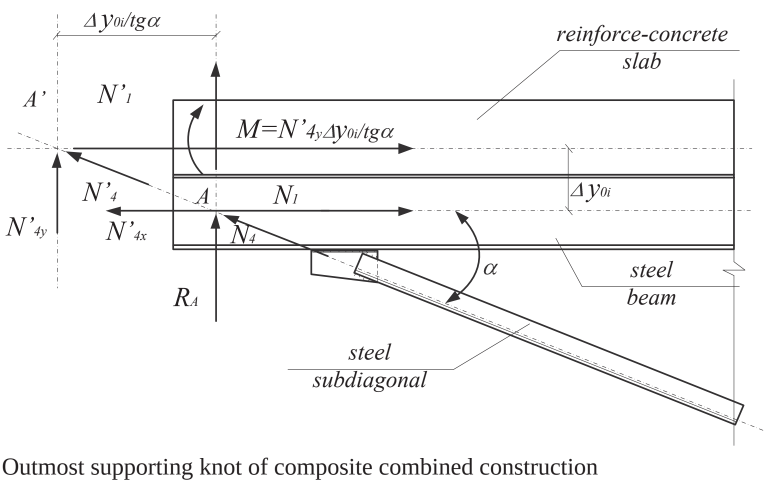

Fig. 6. Outmost supporting knot of composite combined construction

Rys. 6. Węzeł skrajny konstrukcji zespolonej stalowo-żelbetowej

Previous theoretical studies of other authors are undertaken on the example of the steel combined constructions give an opportunity to establish, that change position of mathematical axis of stiffness beam in combined steel-concrete structure considerably increases bearing strength of the combined construction on the whole. In calculating the steel beams of the upper zone, which are combined with reinforced concrete slab, it is necessary to consider the specifics of their work under loading, related to the adopted method of structure manufacturing. Efforts regulation in trussed steel-concrete combined construction is provided by compatible interaction of upper steel-concrete and lower steel parts. At the same time, the stress-deformation state regulation is advisable to carry out simultaneously with technological method; it means stepwise inclusion in the work of the various parts of the structure during the constructing.

For example, show efforts regulation in steel trussed static indefinite combined structure , the upper part of which in the execution of construction works will work as part of steel concrete structure. As the supporting ceiling structures in the middle of the building were proposed steel-concrete combined structures with span of 18 meters, held under the scheme 6 + 18 + 6 m (Fig. 7).

a

b

c

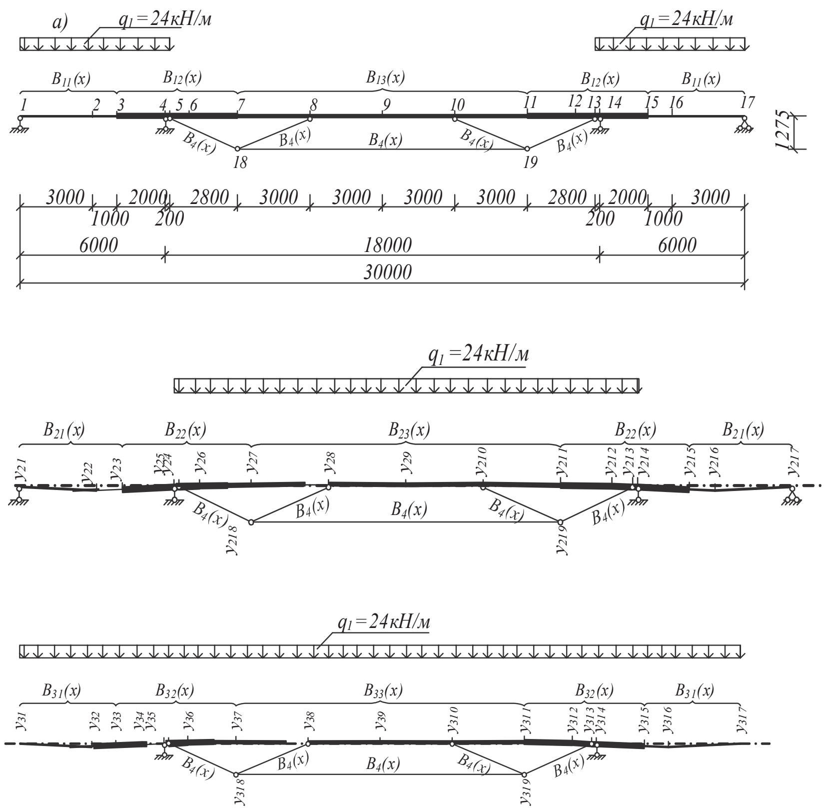

Fig. 7. Loading scheme for continuous combined structure: a – stage I; b – stage II; c – stage III

Rys. 7. Schematy obciążeń konstrukcji zespolonej: a – faza I, b – faza II, c – faza III

At the first design stage, according to the calculation method of the combined single span steel construction with span of 18 m, by step calculation it is found geometric and stiffness parameters of its elements.

The next step was the overlap calculation for continuous triple span scheme of work. As a result of concrete set and concrete strengthening in outmost spans, cross sections of beams in these spans from steel turn into steel-concrete cross sections, whose stiffness characteristics are significantly higher than the original. In this case, the position of the neutral axis of the cross sections in steel-concrete structure is shifting.

Calculation scheme of such a complex structure is a continuous combined steel-concrete and steel construction with variable stiffness along the span, considering changing the vertical position of neutral axis.

The values of the neutral axis position are obtained in the course of previous calculating the vertical displacement of characteristic points of the structure (Table 1 and 2).

The proposed calculative method of steel-concrete combined structures provides in equation of deformations continuity dependence between bending moment and vertical movements. When modifying the neutral axis position, the deformation continuity equations become:

$$\delta_{m,k-2}X_{k-2} + \delta_{m,k-1}X_{k-1} + \delta_{m,k}X_k + \delta_{m,k+1}X_{k+1} + \delta_{m,k+2}X_{k+2} + \\ + \frac{y_{k-1} + y_{k-1}'}{l_1} - \frac{(y_k + y_k')(l_1 + l_2)}{l_1 * l_2} + \frac{(y_{k+1} + y_{k-1}')}{l_2} = 0 \tag{4}$$

where: $y_{k-2}', y_{k-1}', y_k', y_{k+1}', y_{k+2}'$ – vertical displacement determined in previous calculations for typical construction nods.

The deformation continuity equations (2), taking into account the received amendments to the stiffness beam, become:

$$\begin{aligned}
&\delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 + \delta_{14}X_4 + \delta_{15}X_5 + \delta_{16}X_6 + \delta_{17}X_7 - \frac{2y*_1}{l'_1} + \frac{y*_2}{l'_2} = 0 \\
&\delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 + \delta_{24}X_4 + \delta_{25}X_5 + \delta_{26}X_6 + \delta_{27}X_7 + \frac{y_1*}{l_1'} - \frac{y*_2\,(l'_1 + l'_2)}{l_1' x l_2'} + \frac{y_3*}{l_2'} = 0 \\
&\delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 + \delta_{34}X_4 + \delta_{35}X_5 + \delta_{36}X_6 + \delta_{37}X_7 + \frac{y_2*}{l'_2} - \frac{2y*_3}{l'_2} + \frac{y_4*}{l'_2} = 0 \\
&\delta_{41}X_1 + \delta_{42}X_2 + \delta_{43}X_3 + \delta_{44}X_4 + \delta_{45}X_5 + \delta_{46}X_6 + \delta_{47}X_7 + \frac{y*_3}{l_2'} - \frac{y*_4\,(l'_1 + l'_2)}{l'_1\, x l'_2} + \frac{y*_5}{l_1'} = 0 \\
&\delta_{51}X_1 + \delta_{52}X_2 + \delta_{53}X_3 + \delta_{54}X_4 + \delta_{55}X_5 + \delta_{56}X_6 + \delta_{57}X_7 + \frac{y*_4}{l_1'} - \frac{2y*_5}{l_1'} = 0 \\
&\delta_{61}X_1 + \delta_{62}X_2 + \delta_{63}X_3 + \delta_{64}X_4 + \delta_{65}X_5 + \delta_{66}X_6 + \delta_{67}X_7 + y*_6 = 0 \\
&\delta_{71}X_1 + \delta_{72}X_2 + \delta_{73}X_3 + \delta_{74}X_4 + \delta_{75}X_5 + \delta_{76}X_6 + \delta_{77}X_7 + y*_7 = 0
\end{aligned} \tag{5}$$

Table 1. The values of the forces and stresses in the combined structure elements depending on loading stages

Tabela 1. Wartości sił i naprężeń w elementach konstrukcji zespolonej przy różnych fazach obciążenia

stage	№ element		1 2	2 3	3 4	4 5	5 6	6 7	7 8	8 9	9 10	10 11	11 12	12 13	13 14	14 15	15 16	16 17	5 18	8 18	18 19
	*moments, kN*m*	*l*	0	2,194	-6,674	-38,811	-30,267	-16,963	2,857	-3,412	18,188	-3,412	2,857	-16,963	-30,267	-38,811	-6,674	2,194			
		r	2,194	-6,674	-38,811	-30,267	-16,963	2,857	-3,412	18,188	18,188	2,875	-16,963	-30,267	-38,811	-6,674	2,194	0			
	longitude force, kN		0	0	0	0	-44,221	-44,221	-44,221	-91,6	-91,6	-44,221	-44,221	-44,221	0	0	0	0	50,167	-52,972	91,6
	stress, MPa		453	1379	2805	2187	1900	290	799	2559	2559	799	290	1900	2187	2805	1379	453	1032	1090	1885
Stage 1	*moments, kN*m*	*l*	0	5,09143	1,98857	-11,4171	-11,4171	-9,63432	-5,17729	1,50824	1,50824	-5,17729	-9,63432	-11,4171	-11,4171	1,98857	5,09143	5,09143			
		r	5,09143	1,98857	-11,4171	-11,4171	-9,63432	-5,17729	1,50824	1,50824	1,50824	1,50824	-5,17729	-9,63432	-11,4171	-11,4171	1,98857	0			
	longitude force, kN		0	0	0	0	4,15989	4,15989	4,15989	8,61691	8,61691	4,15989	4,15989	4,15989	0	0	0	0	-4,719	4,983	-8,617
	stress, MPa		1052	411	825	825	-725	-470	-172	-224	-224	-172	-470	-725	825	825	411	1052	97	102	177
Stage 2	*moments, kN*m*	*l*	0	4,939	1,785	-11,722	-26,863	-22,591	-14,658	-1,488	-1,488	-14,658	-22,591	-26,863	-11,722	1,785	4,939	1,785			
		r	4,939	1,785	-11,722	-26,863	-22,591	-14,658	-1,488	0,37	0,37	-1,488	-14,658	-22,591	-26,863	-11,722	1,785	0			
	longitude force, kN		29,4221	29,4305	4,86557	29,3220	4,42304	4,44006	4,86557	-22,1126	-22,1126	4,86557	4,44006	4,42304	29,3220	4,86557	29,4305	29,4221	24,86	25,995	46,297
	stress, MPa		561	1529	1595	1453	772	104	141	1054	1054	141	104	772	1453	1595	1529	561	512	535	953
Stage 3	*moments, kN*m*	*l*	0	-2,09	-12,39	-47,38	-38,84	-23,03	2,86	5,61	5,61	2,86	-23,03	-38,84	-47,38	-12,39	-2,09	-2,09			
		r	-2,09	-12,39	-47,38	-38,84	-23,03	2,86	5,61	27,38	27,38	5,61	2,86	-23,03	-38,84	-47,38	-12,39	0			
	longitude force, kN		-0,11	-0,12	-0,27	64,75	28,42	24,39	24,35	-19,21	-19,21	24,35	24,39	28,42	64,75	-0,27	-0,12	-0,11	45,01	-48,18	83,61
	stress, MPa		261	1549	2463	2169	1532	115	289	1782	1782	289	115	1532	2169	2463	1549	261	926	991	1720

Table 2. The values of vertical deflections of combined structure nodes depending on loading stages

Tabela 2. Wartości ugięć pionowych konstrukcji zespolonej w różnych fazach obciążenia

№ element / stage	1 2	2 3	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
stage 0	0	-5,869	-1,694	0	-0,576	-4,226	-18,501	-37,606	-49,1	-37,606	-18,501	-4,226	-0,576	0	-1,694	-5,869	0	-19,621	-19,621
stage 1	0	-13. 4308	-9. 7036	0	0. 7897	3. 4065	6. 0819	4. 2731	3. 0850	4. 2731	6. 0819	3. 4065	0. 7897	0	-9. 7036	-13. 4308	0	2,58	2,58
stage 2	0	0,96	0,65	0	-0,03	-1,01	-8,49	-21,64	-29,02	-21,64	-8,49	-1,01	-0,03	0	0,65	0,96	0	-11,152	-11,152
stage 3	0	1,34	1,83	0	-0,57	-3,56	-13,79	-27,81	-34,67	-27,81	-13,79	-3,56	-0,57	0	1,83	1,34	0	-13,37	-13,37

Offered principle of imaginary hinges introduction in the accepted calculation scheme for static indefinite combined steel-concrete construction is universal. It's application gives an opportunity more simply model the stress-deformation state in the construction elements on the initial design stage. Also, we can include the change of mathematical axis position of stiffness beam. Such approach allows project an economy construction on the whole.

CONCLUSIONS

Calculation of combined steel-concrete structure considering repositioning of mathematical axis shows that the maximum stresses consist up to 75% of allowable; lower level of stresses in steel-concrete structure makes it possible to reduce the size of stiffness beam by 25%.

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NAPRĘŻENIA I DEFORMACJE W ZESPOLONYCH KONSTRUKCJACH STALOWO-ŻELBETOWYCH PRZY ZMIANIE POŁOŻENIA OSI

Streszczenie. Obliczenia zespolonych konstrukcji stalowo-betonowych przeprowadza się w sposób fazowy. Przed uzyskaniem pełnej wytrzymałości betonu ciężar własny przejmuje konstrukcja stalowa, po zakończeniu procesu dojrzewania betonu konstrukcja pracuje jako zespolona. W artykule omówiono metodę fikcyjnych przegubów przy obliczaniu naprężeń elementów konstrukcji przy zmianie jej sztywności giętej. Opisana metodyka znajduje zastosowanie na przykład przy analizach konstrukcji zespolonych, z uwzględnieniem fazowości ich pracy.

Słowa kluczowe: prężna oś, konstrukcje stalowo-betonowe, równanie ciągłości deformacji

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