

MODAL ANALYSIS QUALITY IDENTIFICATION METHODOLOGY APPLIED TO WALL-ELEMENTS

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ABSTRACT

New recommendations and norms in civil engineering show a need to control the quality of wall elements; the quality control demands guidelines to fit requirements of mass-production of wall elements. This article develops an approach which applies advanced calculating techniques used for wall-structural analysis. The paper proposes a diagnostic methodology which can be added to regulations and standards based on experimental modal analysis methodology.

Key words: coherence function, experimental modal analysis, stabilization diagram, wall element

INTRODUCTION

Existing civil constructions and masonry structures are subjected to dynamic effects caused by the environment (wind, earthquakes and sea waves) and vibration or trembling events (explosions). Tremblings in buildings decrease living comfort, has an influence on the people working there and threatens the construction safety level – trembling events cause dynamic load and can generate catastrophic destruction (Liss, Żółtowski, Żółtowski, Sadowski & Kuliś, 2016).

The structural engineering recognizes the need of improving the methods of determination of wall elements quality by means of wall safety assessment coefficients defined by the structural norm, i.e. PN-B-03002 standard (Polski Komitet Normalizacyjny [PKN], 2007). It requires establishing the partial coefficients of wall elements, called safety coefficients, to classify category of works performing on civil sites (PKN, 2007). Currently, wall element quality control is made manually based on visual inspection during production process; this method

is subjective and does not evaluate the technical state of an element.

In this paper, a technique for assessment of damage level of wall elements has been adopted, based on non-intrusive tests through the measurement of sets of signals with use of the Experimental Modal Analysis (EMA) techniques. The EMA is applied directly to the tests of wall elements based on measuring the excitation and response of a dynamic system. The EMA has been widely documented, and the modal dynamic properties can be identified using standard identification methods. The methodology proposed for this study includes:

- (i) applying an excitation to a wall element with an impact hammer, and recording the excitation/response signals by means of a data acquisition system;
- (ii) processing of the recorded signals through EMA-based techniques; the modal properties are identified by using:
 - the Least-Square Complex Exponential (LSCE) method to calculate the Stabilization Diagram (SD) (Cempel, 2003),
 - the Coherence function (Coh_{ij}).

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The proposed methodology corresponds to assessment models combining the numerical and symptomatic models to evaluate the critical levels (Żółtowski & Żółtowski, 2014). The developed inspection methodology is feasible to be applied to wall elements, since it uses sensors (force sensor, accelerometers) of easy installation and operation, while they record reliable signals. The proposed approach can be used for better understanding the behavior of constructions, which allows optimizing the projection and assessment of critical states.

MATERIAL AND METHODS

The EMA is used to determine the behavior of the wall elements under given dynamic effects (Uhl, 1997; Avitabile, 2015). Consider a square matrix, A — of real numbers having a size of $n \times n$, eigenvalues λ_r and corresponding eigenvectors $\{\varphi\}_r$ with $(r = 1, 2, \dots, n)$; the $\{\varphi\}_r$ family consists of independent vectors. The matrix of λ_r can be expressed in the form $\Lambda = \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_n]$, and the matrix of $\{\varphi\}_r$ as $[\psi] = \text{diag} [\{\varphi\}_1, \{\varphi\}_2, \dots, \{\varphi\}_n]$.

The decomposition of eigenvectors produces the equation (Żółtowski & Martinod, 2015) $A] = [\psi] [\Lambda] [\psi]^{-1}$.

The equation shows that A — may be expressed as a diagonal matrix in the form $[\Lambda] = [\psi]^{-1} [A] [\psi]$. For all range, the only solution satisfying the condition that λ_r and its corresponding non-null $\{\varphi\}_r$ exist is when $([A] - \lambda_r [I])\{\varphi\}_r = \{0\}$. The polynomial's roots are λ_r of $[A]$. $\lambda_r = 0$ indicates that there is no vibration, and a body is rigid and fixed to the ground. If $\lambda_i = \lambda_r$, it means that there are identical modal shapes, a phenomenon that occurs frequently in symmetrical structures.

The EMA-LSCE method

The EMA associated to the LSCE method assesses the global modal parameters: natural frequency ($\sqrt{\lambda_r}$) is equivalent to damping ratio. The LSCE method determines the Impulse Response Function (IRF) relation in a Multiple Degrees of Freedom (MDoF) sys-

tem. The IRF can be derived from the inverse Fourier Transform (FT) for a Frequency Response Function (FRF) through the Power Spectral Density (PSD), the Random Decrement (RD) process, the inverse Laplace transform or other methods (Uhl, Sękiewicz, Hanc & Berczyński, 2005).

The inverse Laplace transform of the transfer function of an MDoF system is the IRF h_k . This gives as a result a series of equally spaced time intervals $k\Delta (k = 0, 1, \dots, 2n)$, and then it is possible to express IRF (Żółtowski & Martinod, 2016)

$$h_k = \sum_{r=1}^{2n} {}_r A_{ij} z_r^k; \quad \text{with} \quad z_r^k = e^{s_r k \Delta} \quad (1)$$

where

$${}_r A_{ij} = \varphi_{ir} \varphi_{jr} \quad (2)$$

This expression is the product of the i th and j th elements in the modal shape r th, $\{\varphi\}_r$, and named as modal constant. The values in the series belong to the real numbers even if the residues and the roots s_r are complex values. It is possible to demonstrate that all imaginary parts will cancel each other because of the complex conjugates for both expressions: ${}_r A_{ij}$ and s_r . The next step is to estimate the roots and the residues for the sampled data. This solution is aided by the conjugate of the roots s_r , therefore z_r . Mathematically, this means that z_r are the roots of a polynomial with only real coefficients

$$\beta_0 + \beta_1 z_r + \beta_2 z_r^2 + \dots + \beta_{2n} z_r^{2n} = 0 \quad (3)$$

$$\sum_{k=0}^{2n} \beta_k h_k = \sum_{r=1}^{2n} {}_r A_{ij} \sum_{k=0}^{2n} \beta_k z_r^k$$

This equation is known as the *Prony* equation. The coefficients can be estimated by the IRF values. An Auto-Regressive (AR) model is constructed by the relation between poles and residues, which are processed to perform the estimation of the poles given, as they provide information about the quality of the data and the requirements concerning computational resources (a model of higher order implies a greater

processing cost) (Williams, Crowley & Vold, 1985). The AR model solution allows to define a polynomial which, among its roots, has the complex ones as well; having established the roots (equivalent to the natural frequencies, and the damping rates), the residues can be derived through the AR model and then the modal shapes $\{\varphi\}_r$ can be obtained (Żółtowski, 2012).

With the calculation of the system poles, it is possible to build the Stabilization Diagram (SD), which allows graphically represent the poles of a system when it is excited in one point (reference point) and the measurements are made in another one (responses) (Żółtowski, Żółtowski & Liss, 2016). The estimated poles for certain frequencies create stable vertical lines. The vertical lines are generated in the characteristic frequencies of the system; then, it is possible to affirm that the pole identifies modal parameters. The SD exposes the characteristics of the poles, which are presented in code with alphanumeric characters: stable pole (*s*), stable vector and modal vector frequencies (*v*), stable vibration and softening frequencies (*d*), stable vibration frequency (*f*), and unstable pole (*o*). Once the poles have been selected it is possible to estimate the vibration shape (Żółtowski & Żółtowski, 2015a).

EMA Coh_{xy} -function

Coh_{ij} function finds the estimation of the coherence magnitude and an excitation signal $i(f)$ and response signal $j(f)$ in the frequency domain, using Welch's averaged, modified periodogram method. The magnitude squared coherence is a function of frequency domain with a range of 0–1 values – indicating how well $i(f)$ corresponds to $j(f)$. The coherence is a function of the power spectral density P_{ii} , P_{jj} of $i(f)$ and $j(f)$ and the cross power spectral density P_{ij} , then

$$Coh_{ij}(f) = \frac{|P_{ij}(f)|}{P_{ii}(f) \cdot P_{jj}(f)}$$

$Coh_{ij}(f)$ is calculated with a periodic Hamming window of length to obtain eight equal sections of $i(f)$ and $j(f)$.

The research has been conducted with two identical sets of wall elements, the properties of all bricks were similar: (i) length, 250 mm; (ii) width, 120 mm;

(iii) height, 65 mm; and (iv) mass, 3.5 kg; provided by a recognized brick producer in Bydgoszcz (Poland).

One set had been damaged during the manufacturing process and will be called fault wall-elements W_F . The second set has approved quality, representing the nominal structural behavior, i.e. the second set shows the reference values W_R . Each wall-element was tested along the principal axes: (i) x – longitudinal; (ii) y – lateral; and (iii) z – vertical.

The measurement point is important as it has influence on the results of modal investigations (Allemang & Phillips, 2004; Avitabile, 2015). The accelerometers should be fixed in such way that they will not influence the arrangement performance (Stepinski, Uhl & Staszewski, 2013); as well as they should be fixed in characteristic places of the wall element. The wall elements must be fixed to a correct assemblage according to the input perturbations. Perturbations must be performed on an object in normal operation; during experimental realization, the mounting must have the correct boundary conditions to get realistic DoF (Żółtowski & Żółtowski, 2015a). The excitation-response performance has been recorded in a DAQ system by means of a set of single-axial piezoelectric accelerometers, PCB-352C68-ICP model. The signals in the time domain from DAQ system were exported to a real-time signal software package.

The obtained results have been analyzed with an algorithm written in MATLAB programming language: the Virtual In Operation Modal Analysis (VIOMA) (Żółtowski & Żółtowski, 2015b) and the Computer System of Identification Investigations (SIBI) (Żółtowski, 2014). Examples of datasets recorded, the data recorded from W_F and W_R are shown in Figures 1 and 2.

An estimator value must be defined to synthesize the SD and Coh_{ij} information and identify the technical state of the wall elements. The estimator defining criterion is the computational resource requirements (low processing cost), the selected estimator is the area under the SD and Coh_{ij} functions, $\{SD, Coh_{ij}\}_{(Area)}$. Figure 2 shows recorded signal estimator and shows a clear difference in identification of quality of the wall elements, due to reference difference relative to the fault wall elements at the range of 88.3% and 82.7% to the estimator $SD_{(Area)}$ and $Coh_{ij(Area)}$, respectively.

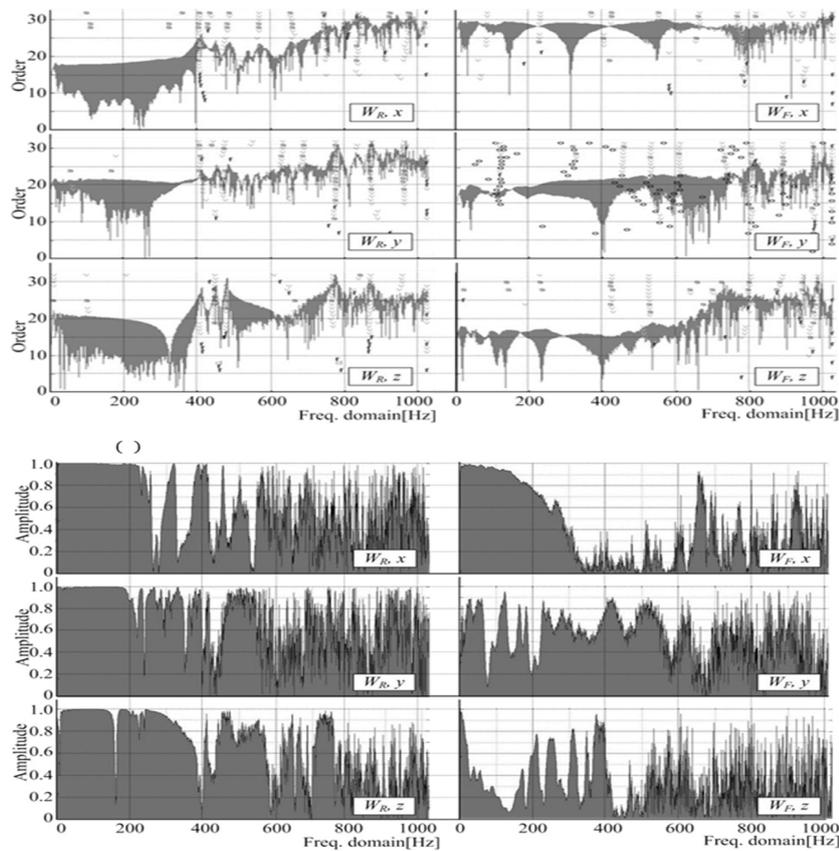


Fig. 1. Stabilization diagrams and coherence functions of recorded signals

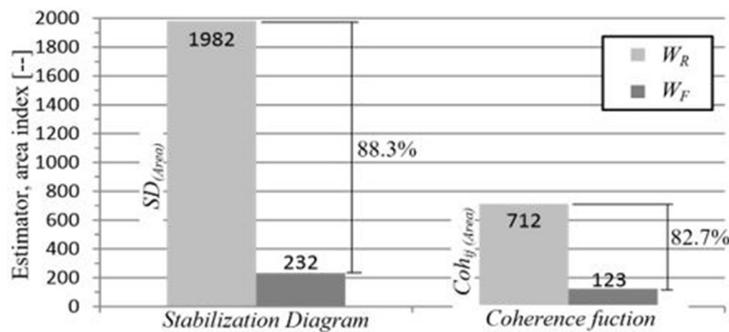


Fig. 2. Recorded stabilization diagrams, and coherence functions

CONCLUSIONS

The introduced investigation results show the existing possibility of distinguishing changes in material properties. The investigation confirmed that the application is useful as it makes possible to create SD and $Coh_{ij}(f)$

functions. The data obtained from these diagrams and functions allow assessing the material state by comparing their fitness.

From the obtained results, the following statement was made:

- for a full fit wall element W_R , a stable pole can be generated in the x direction at a stable level of 420 Hz. In case of a fault wall-element W_F , it was not possible to generate a stable pole in the SD. The situation is the same when comparing x and y SD. This means that for W_R , a stable pole can be generated in the y axis at a stable level of 790 Hz; in case of a W_F , it is not possible to generate a stable pole in the SD;
- for z -axis, it was not possible to generate stabilization diagrams neither for good or damaged materials. The studies performed according to axis z did not provide any answers, therefore, further investigation should be performed only for x -axis and y -axis.

The estimator value $\{SD, Coh_{ij}\}_{(Area)}$ represents an appropriate index to identify the technical state of wall elements and to define the quality control process of wall elements. The $\{SD, Coh_{ij}\}_{(Area)}$ can be directly normalized to get an index into the scale defined by a national structural standard.

The proposed methodology is useful whereas the data logging allows getting information in terms of commercial operation, avoiding measurements requiring downtime for the system inspection.

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METODA IDENTYFIKACJI JAKOŚCI ELEMENTÓW ŚCIENNYCH Z ZASTOSOWANIEM ANALIZY MODALNEJ

STRESZCZENIE

Współczesne zalecenia wskazują na konieczność kontroli jakości wykonania elementów ściennych; kontrola jakości wymaga odpowiednich wytycznych, aby dopasować się do wymagań bieżącej masowej produkcji elementów ściennych, a następnie norma konstrukcyjna uznaje potrzebę udoskonalenia metod identyfikacji rzeczywistej jakości elementów. Przedstawiona praca została opracowana z zastosowaniem podejścia wykorzystującego zaawansowane techniki obliczeniowe stosowane do analizy strukturalnej w inżynierii lądowej, skoncentrowanej na ocenie stanu technicznego elementów ściennych. W artykule zaproponowano wykorzystującą eksperymentalne techniki analizy modalnej metodologię diagnostyczną, którą można uwzględnić w obowiązujących przepisach i normach.

Słowa kluczowe: funkcja koherencji, eksperymentalna analiza modalna, diagram stabilizacyjny, element ścienny